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TRACTIVE RESISTANCE OF COHESIVE CHANNELS

Irving S. Dunn,[1] A. M. ASCE

SUMMARY

A method of estimating the tractive resistance of cohesive channel beds is proposed. The method is based on information obtained from simple soil tests (Atterberg Limits, Particle Size Analysis, Vane Borer Tests). The study was made for soil taken from cohesive channels in Colorado, Nebraska, and Wyoming.

INTRODUCTION

A prime requisite of good canal design is the choice of a channel cross section and slope which will carry the necessary amount of water without an appreciable amount of scour or deposition. Failure to accomplish this objective has resulted in high maintenance costs caused by channel obstructions, clogged diversion works, eroded surface drainage systems and undermined foundations.

The main guide in this phase of canal design has been an accumulation of data, gathered mostly from experience, which relates the stability of a channel in a specified material to the maximum permissible average velocity. The soil materials in this relationship have been described by such terms as silty clay, hardpan, loam, etc. These terms are very general and are not adequate to fully describe the properties of the soil with respect to their resistance to erosive forces.

In recent years, investigators have produced a considerable amount of information on scour and the rate of transportation of sediment from sandy linings. This information has been mainly related to the grain size characteristics of the sandy material.

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There is a need to extend this information into the range of cohesive fine grained soils with the objective of discovering a set of soil characteristics which will be useful in estimating resistance to erosion for soils in which waterways are proposed. The present study is an attempt to satisfy this need through a laboratory investigation of the tractive resistance characteristics of some cohesive soil samples taken from canal beds.

Experimental Procedure

The effect of soil properties on tractive resistance was investigated by the writer by measuring the hydraulic shear stresses necessary to cause erosion of soil samples in which the cohesion was varied by applying different degrees of preconsolidation.

Collection of Soil Samples

Several channels, with beds ranging from sand to thick silty clay, were selected in Nebraska, Wyoming, and Colorado. The sites selected were comparatively long, straight reaches, and were relatively free from large vegetation. Soil samples were taken with a post hole digger from the top surface of the bed (0-5 in.). Disturbed soil taken from the upper five inches of bed is not representative of the soil at the surface, but this is not important since this study was undertaken to relate soil resistance to hydraulic shear stresses and not for the purpose of investigating the stability of the channel beds from which the samples were taken. The soil was oven dried and passed through a No. 10 U. S. Standard sieve. The samples contained very little material retained on this sieve, the maximum amount for any sample being three per cent. This coarse soil fraction was not used in the samples for the laboratory investigation.

Shear Strength Apparatus

A vane borer was used to measure the shear strength of the soil. The vane had four flanges and was four inches high and two inches in diameter. The vane and torque assembly are shown in Fig. 1. The upper surface of the vane was kept above the surface of the soil and if it is assumed that the shear stress at the time of failure was uniform over the bottom and sides of the sheared cylinder then

$$M_v = S_v\left(\frac{\pi d_v^2 h_v}{2} + \frac{\pi d_v^3}{12}\right) \tag{1}$$

in which M_v = applied torque in lb in.

S_v = unit shear strength in lbs/in.^{2}

d_v = diameter of vane in inches.

h_v = height of sheared cylinder in inches.

By substituting the dimensions of the apparatus used in these experiments into Eq. (1), we obtain

$$S_v = \frac{0.715F}{h + 0.333} \tag{2}$$

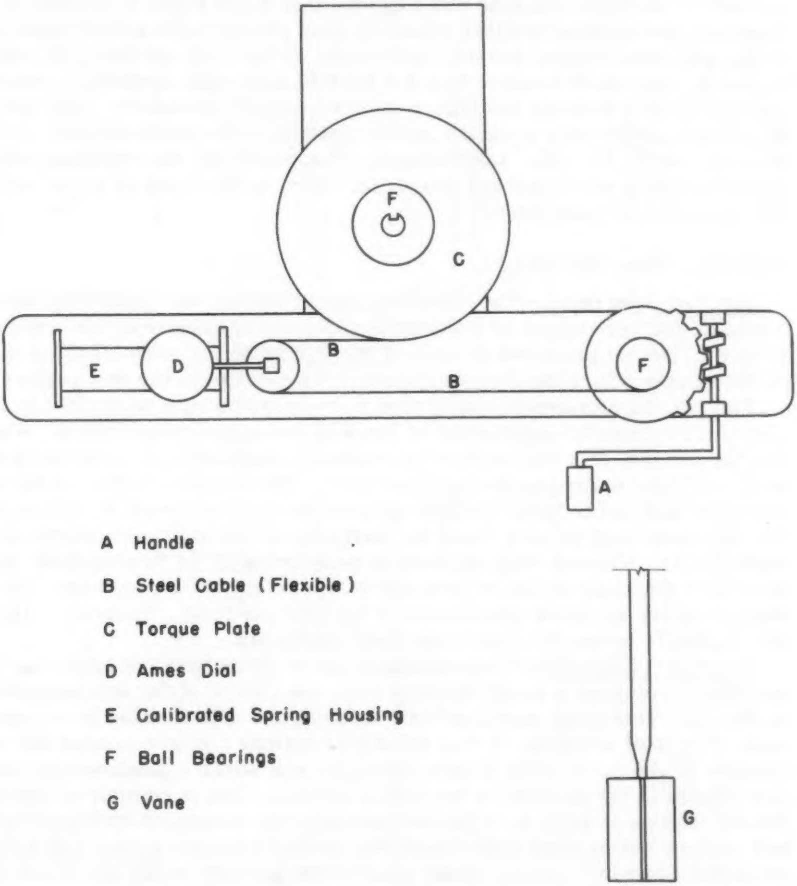

Figure 1 Vane Test Apparatus

in which F = tension in the cable attached to the torque plate (lbs).

In these experiments a question may exist concerning the effect of the size of the sample container on the value of vane strength. The container used had a diameter of 6.25 inches and the diameter of the vane was two inches. If the walls of the container are too close to the failure cylinder, a rigid soil bridge may be built up between the vane and the container during the shear test. This would increase the value of the lateral stress (σ), and provide an erroneous value of S_v. An examination of the shear strength obtained for sand samples indicates that a lateral pressure of about one $lb/in.^2$ was present at the time of failure. This value is greater than could be expected near the

surface in unrestricted sand and suggests that some error is present in S_V. However, the critical tractive stress in sand proved to be almost independent of S_V, and these errors are not significant. In the clay samples, the values of $\sigma_V \tan \phi_V$ was much smaller than for sand because clay presents a much less rigid structure between the failure cylinder and the container. The fact that ϕ_V is less in clay than in sand also contributes to the insignificance of the term $\sigma_V \tan \phi_V$ for clay. Consequently, it appears that the container used was large enough to eliminate any important effect on the value of shear strength measured by the vane borer.

Hydraulic Shear Stresses

Jet Test Apparatus.—The disturbing shear stress was applied by means of a submerged vertical jet of water directed perpendicularly at the soil surface (Fig. 2). The jet produced erosion of the soil at points away from the center of the sample where the flow was essentially parallel to the soil surface.

The results obtained by use of this test apparatus may be applied to field channels because the application of force is the same in both cases. Water flowing parallel to a soil surface produces a combination of turbulent form drag and viscous drag on the soil particles. The characteristics of turbulence in the jet test and in field channels are not similar; however, it will be shown that the form drag is very small for particles of the size encountered in cohesive soils. Viscous drag depends on properties of the flowing fluid and soil which are the same in the jet test and in the field; namely, the viscosity of the water, and the frictional properties of the soil particles. Therefore, the test may logically be used to represent field conditions.

Jet Test Calibration.—The maximum shear stress and the beginning point of erosion occurred a small distance from the center of the soil sample (point A, Fig. 2). This point was found by observing the action of the jet of water on each of the soil samples. It was found that neither changes in head (H) nor changes in elevation of the nozzle above the soil surface produced any noticeable change in the position of the initial erosion. The magnitude of the shear stress existing at point A on the soil surface was measured by replacing the soil surface with a steel plate which was coated with soil grains and which contained a one inch square shear plate in the position of the maximum stress (Fig. 2a).

The force on the shear plate was measured using the variation in the plate voltage of the transducer tube (RCA 5734-triode tube) which was mounted on the assembly. The tube was calibrated with the knife-edge balance shown in Fig. 3.

Typical Test Description

In order to clarify the test procedure, a description of a typical test follows.

A sample of soil which had been oven dried and passed through a No. 10 U. S. Standard sieve was thoroughly mixed with water and puddled into a metal container (A in Fig. 4). The moisture content was above both saturation and the liquid limit, but was not high enough to allow segregation of the soil particles. The soil was consolidated between porous plates for three days. The load and top porous plate were then removed and the lucite cylinder (B) was clamped into position and filled with water. The soil was allowed to come to equilibrium under zero load for two days. This time was considered sufficient

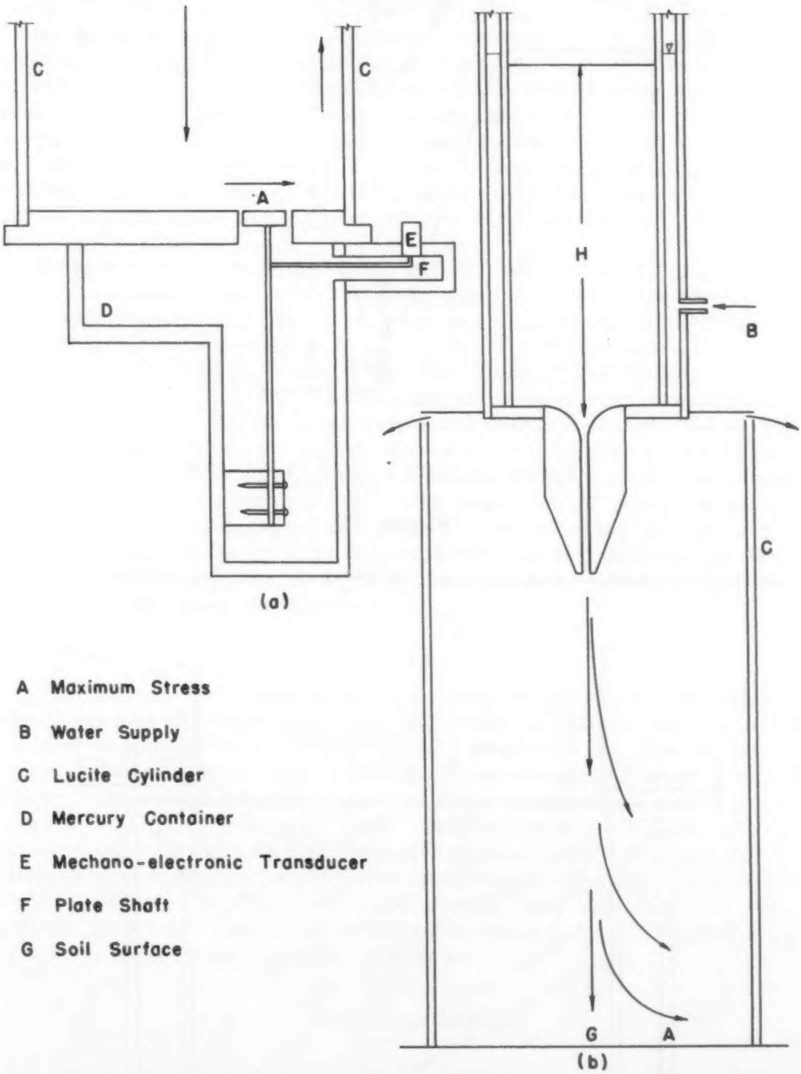

Figure 2 Jet Test Apparatus and Calibration Mechanism

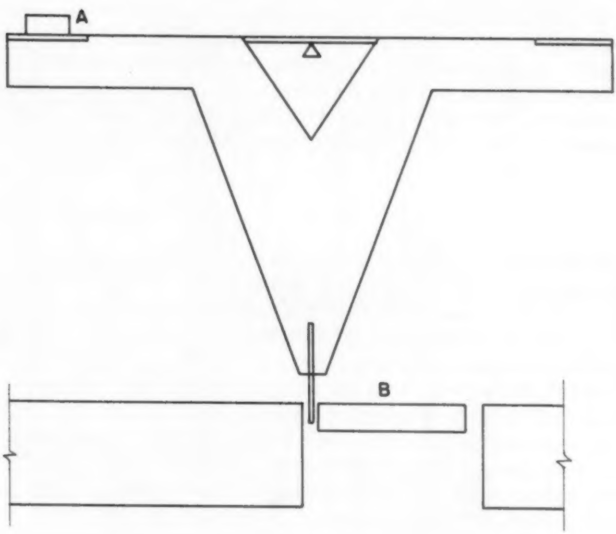

Figure 3

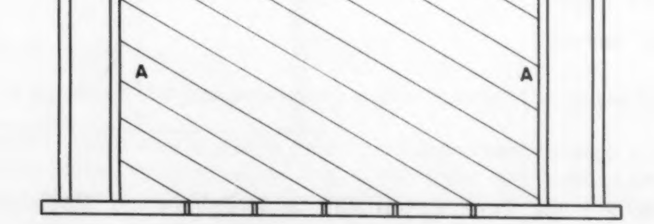

Figure 4

to reduce excess hydrostatic pressures in the soil to a negligible value. The jet was then positioned above the soil sample and the head of water on the nozzle was slowly increased. As the head was increased the water first removed material from the surface of the soil which had been disturbed by removing the upper porous plate. With each additional increase in H, a small additional amount of soil was carried off the surface followed by clearing of the water in the container. When H reached the critical value, the rate of erosion increased, the water became cloudy, and no subsequent clearing occurred. The critical point was definite and reproducible in the clay samples. The head was then decreased to determine whether the critical tractive force became smaller after the initial disturbance of the soil surface. It was found that the erosion of the surface produced no significant change in the head necessary to continue erosion.

The sand samples presented more difficulty than the clay samples in the choice of a critical value by this method, since some particles were in motion at the lowest possible heads. An attempt was made to choose the critical value of H at a time when general motion of the particles on the surface existed. The results of these observations were reproducible but were not nearly as well defined as the critical conditions for the more cohesive samples.

After removal of the jet apparatus, the vane borer was inserted vertically into the soil until the bottom of the vane was two inches from the bottom of the soil sample and the depth of insertion was measured. The vane was rotated at a rate of 0.1 degrees per second until a peak strength was obtained.

The above test procedure was repeated on the same soil sample with different consolidation loads (ranging from 25 to 300 lbs/ft^2) until enough data was gathered to determine the variation of hydraulic shear strength (S_h) with changes in vane shear strength (S_v).

Field Tests

In place shear tests were made on the beds of the canals from which laboratory samples had been taken. The tests were performed with the vane borer mounted on a tripod base. The testing took place with water in the canal. Several tests were made and an average shear strength obtained at each location. These tests furnished values of vane strength which were used to determine the in place resistance of the canal beds to erosion. The laboratory samples were not entirely representative of canal conditions since the samples were taken from the top five inches of the bed instead of only from the surface material. The correlation between field and laboratory results, however, is not pertinent to the conclusions drawn in this study, and is reported merely as an additional item of interest.

Theoretical Analysis

A functional relationship between the disturbing and resisting forces for a cohesive soil may be obtained by an examination of the equilibrium of a flake of soil at the surface with erosion impending (Fig. 5).

The eroded flake may consist of a floc of many soil grains. The force tending to cause erosion is

$$F_d = T_t A \tag{3}$$

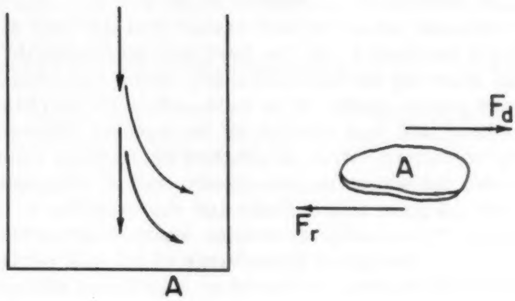

Figure 5 Forces Acting on an Eroding Flake of Soil.

where

$$T_t = T + T^1 \tag{4}$$

in which T^1 = stress caused by turbulent form drag.

A = area of the eroded flake of soil.

T = stress caused by viscous drag.

White,(1) after an examination of the work of Nikuradse and Fage, states that of the two effects—viscous drag and turbulent form drag—viscous drag predominates in flows in which

$$\frac{V_* d}{\nu} = \frac{d\,(T_t/\rho)^{\frac{1}{2}}}{\nu} < 3.5 \tag{5}$$

in which V_* = friction velocity.

T_t = average shear stress in gr/sq cm.

= density of water in gr sec^2/cm^4.

= kinematic viscosity in cm^2/sec.

= diameter of particle in cm.

For the cohesive soils tested by the writer, the value of the right-hand expression in Eq. (5) was in the range 1.45 to 5.7. Even though some of the values exceeded 3.5, it is reasonable to assume that almost all of the disturbing force in these tests was accounted for by viscous drag. This assumption was borne out by observation that a change in shape of the soil surface caused by erosion produced no change in the head of water necessary to continue erosion. If form drag were important, the critical head of water would decrease when the surface of the soil eroded. Therefore, the disturbing force in Eq. (3) becomes

$$F_d = TA \tag{6}$$

If we assume that Coulomb's shear strength equation applies in the case of hydraulic stress, then the resisting force is

$$F_r = (\sigma \tan \phi_h + c_h)A = S_h A \tag{7}$$

in which S_h = unit strength of soil resisting hydraulic shear stresses.

σ = normal intergranular stress on surface.

ϕ_h = angle of friction resisting hydraulic stresses.

c_h = cohesion resisting hydraulic stresses.

Two questions arise with respect to Eq. (7).

1. What effect does the curvature of the flow in the jet test have on σ? The impulse momentum principle shows that an average vertical force of 1.44 lbs/sq ft was present on the soil surface as a result of flow curvature at the highest head used in the experiments. At the position of the flake of soil the normal force due to curvature was approximately

$$\sigma_c = \frac{1.44\,KH}{H_{max}} \text{ lbs./sq. ft.} \tag{8}$$

in which K = stress concentration factor.

2. Does the cohesion of the soil act in the same way in resisting the stresses caused by the vane borer and those caused by the jet test? The forces of attraction between soil grains are responsible for resistance to shear caused by the vane borer and also for resistance to shear caused by the flow of water. However, since the shear mechanism is different in these two cases, it is possible that the relationships between attractive force and these two shear phenomena are of different forms. This difference can be expressed as a function of the type of soil in the following way.

$$C_h = S_v f(P) + f_1(P) \tag{9}$$

in which f(P) and f_1(P) are functions of the plastic properties of the soil.

With the corrections given above, Eq. (7) for the resisting force in the jet test becomes

$$\begin{aligned} F_r &= A\left(\frac{1.44\,KH}{H_{max}} \tan\phi_h + C_h\right) \\ &= A\left[\frac{1.44\,KH}{H_{max}} \tan\phi_h + S_v f(P) + f_1(P)\right] \end{aligned} \tag{10}$$

and equating this to the disturbing force in Eq. (6),

$$T_c = \frac{1.44\,KH}{H_{max}} \tan\phi_h + S_v f(P) + f_1(P) \tag{11}$$

Eq. (11) states that the critical tractive stress of the soil is a linear function of the shear strength obtained with the vane borer. The first term of the equation represents a contribution to resistance caused by flow curvature in the jet test. This term is of small magnitude and since ϕ_h is less for clay than for sand, and since H is greater for clay than for sand, the term has the

effect of a small constant added to the true tractive resistance (S_h) of each soil. This effect appears only in the jet test and therefore the first term of Eq. (11) should not be included when using the equation to determine tractive resistance of soils in the field.

Discussion of Results

The data from the experiments performed by the writer and explained in the section on experimental procedure are presented in Fig. 6. Each line on this graph represents the experimental points for one soil. The lines are straight to correspond with Eq. (11) and appear to converge at a common point to the left of the graph and to exhibit an inclination θ with the horizontal which is independent of the vane strength. The equation of the family of lines is

$$T_c = 0.02 + \frac{S_v \tan\theta}{1000} + 0.18 \tan\theta \qquad (12)$$

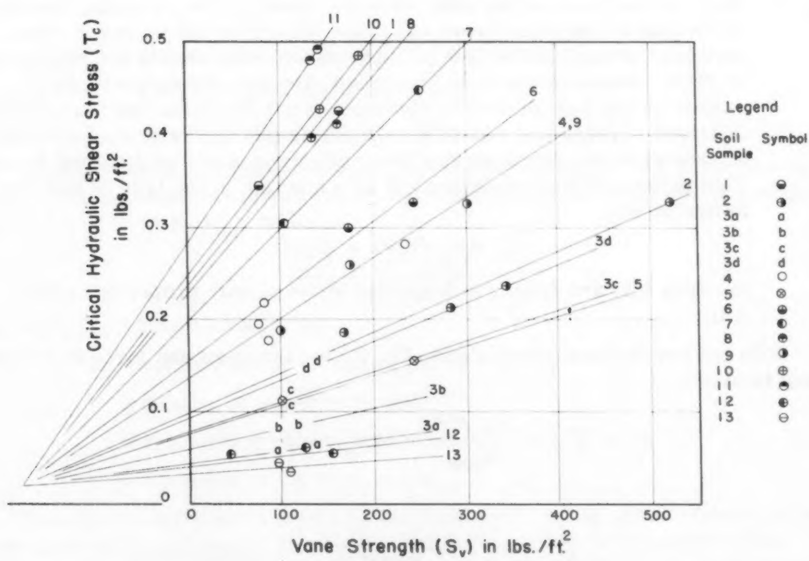

Figure 6

The lines are not valid in the range $0 < S_v < 60$ lbs/ft^2, and only represent the extension of the lines through the region investigated. The Y intercept of the lines does not represent the value of critical tractive stress for the case of zero vane strength. If the vane strength were zero, one could assume that the value of T_c would decrease for decreasing particle size, and this is not the case in Fig. 6. An examination of the magnitude of vane strength in actual channels shows that this region is only important for materials which have been recently deposited and have not had the opportunity to gain cohesive strength. If the flow is in such quantity as to make erosion a problem, then deposition of very fine materials will not occur.

Eq. (12) may be compared term for term with Eq. (11) and it becomes apparent that the functions f and f_1 are both related to the soil properties in the same way, namely through $\tan\theta$. The first term on the right hand side of each of Eqs. (11) and (12) is due to the curved flow in the jet test and it will be included in the remarks in this section but it must be subtracted from the critical tractive stress obtained in the jet test in order to obtain the value of resistance which the soil will exhibit under field conditions.

The values of θ for the soils tested are tabulated in Table 1. Eq. (12) may be used to estimate the tractive resistance of a soil if S_v can be measured in a field test with the vane borer, and if θ can be expressed as a function of the soil properties by using a suitable classification system. There are many such systems to choose from in the science of Soil Mechanics, and it is not difficult to find one which will describe the soil samples taken from cohesive channels.

Due to the similarity in source regions and weathering agents for the soils tested, θ may be expressed as a function of the particle size distribution or of the plasticity of the soil. Several different methods of describing the size distribution and plasticity are presented here in order to provide as much information about these soils as is possible.

Effect of Soil "Fines" on Tractive Resistance

The physical properties of soils become more dependent on surface area and less dependent on weight as the size of the particles decreases. If fine particles are added to a soil, it is reasonable to expect that the cohesive properties will be increased. The amount of fine material, therefore, should be an indication of the resistance to tractive force. Fig. 7 shows the relationship between θ and the per cent of silt and clay (U_f) in the soils tested by the writer. A particle size of 0.06 mm was selected as the boundary between silt and sand for purposes of finding numerical values of U_f.

The circles in Fig. 7 represent natural channel materials and the letters represent four samples of synthetic soils formed by mixing Ottawa sand and Wyoming bentonite. It is notable that both types of soils fit into the correlation in a very satisfactory manner.

The equation of the straight line in Fig. 7 is

$$\theta^\circ = 0.6(U_f) \tag{13}$$

in which U_f = per cent of particles smaller than 0.06 mm by weight, and substituting this into Eq. (12) gives

$$T_c = 0.02 + \frac{(S_v + 180)}{1000} \tan(0.6\,U_f) \tag{13a}$$

Table 1.--Properties of the Soils Tested

Soil	θ°	U_f	PI	t	Mean Size M_ϕ	Mean Size mm	Stan. Dev. σ_ϕ	Skew K_ϕ
1	50.5	69.0	11.1	0.817	5.50	0.022	2.60	0.35
2	24.5	35.0	0	0.507	3.80	0.072	2.58	0.50
3a	7.0	12.5	—	0.209	1.60	—	—	—
3b	12.0	18.0	—	0.291	1.65	—	—	—
3c	18.0	26.0	—	0.393	1.70	—	—	—
3d	22.5	44.0	—	0.592	2.00	—	—	—
4	33.0	41.0	0	0.582	3.67	0.078	1.70	1.75
5	17.5	31.0	0	0.483	3.64	0.081	1.48	2.49
6	37.0	46.0	2.5	0.608	4.73	0.038	1.70	1.67
7	44.5	78.0	8.8	0.872	5.95	0.016	2.27	1.10
8	49.5	81.0	13.3	0.890	6.05	0.015	2.22	0.93
9	33.0	56.0	3.5	0.705	5.25	0.026	1.80	1.67
10	52.0	88.0	11.2	0.957	6.10	0.014	1.80	1.48
11	56.0	95.0	15.6	0.985	6.12	0.014	1.61	1.49
12	6.5	10.0	0	0.189	2.85	0.139	0.65	2.70
13	4.0	5.0	0	0.074	2.53	0.173	0.37	3.40

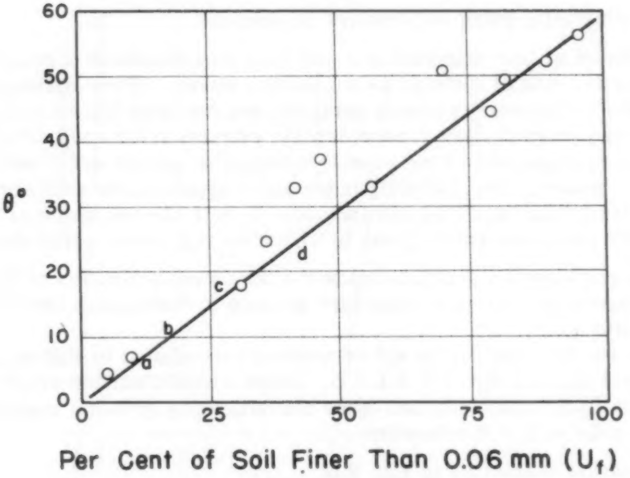

Figure 7

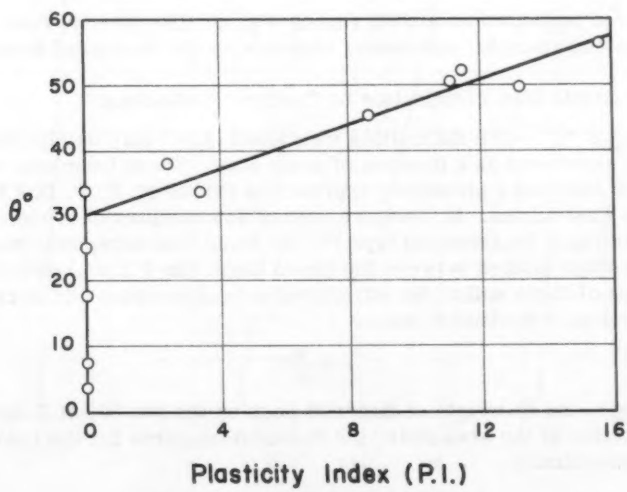

Figure 8

This method will not detect differences in T_c for soils with a very small percentage of sand.

Effect of Plasticity Index on Tractive Resistance

The amount of fine material in a soil may sometimes be a poor indication of the effectiveness of the clay as a cohesive binder. Some extremely fine grains such as pulverized quartz particles exhibit very little cohesion while natural clay particles show a considerable attraction for each other. The plasticity index has often been used to distinguish plastic differences in soils such as the above. The plasticity index and θ appear to be well correlated for the soils in this investigation (see Fig. 8) with the exception of the sandy soils. Here there are two regions in which the P.I. is not particularly useful.

1. The position of the points below $\theta = 30^0$ is not a function of the P.I. A different system must therefore be used to distinguish between non-plastic soils.
2. The value of the P.I. is not accurately reproduced by different investigators in the range $0 < \text{P.I.} < 5$. These two difficulties affect many of the soils in this study and other classification systems must be used for soils with P.I. below five.

The equation of the line in Fig. 8 is

$$\theta^\circ = 30 + 1.73\ P.I. \tag{14}$$

and substituting this into Eq. (12) gives

$$T_c = 0.02 + \frac{(S_v + 180)}{1000} \tan(30 + 1.73\ P.I.) \tag{14a}$$

This curve is plotted as a solid line in Fig. 9. The other curves in Fig. 9 are recommendations for ephemeral channels by the Bureau of Reclamation.[2]

Effect of Particle Size Distribution on Tractive Resistance

Dos Santos' "t".—The difficulties mentioned above may be eliminated if the P.I. can be expressed as a function of grain size. There have been many attempts to do this and a promising approach is that of M. P. P. Dos Santos, of Portuguese East Africa. M. Santos analyzed 353 samples of cohesive soils of different origin, location and type.[3] He found that acceptable mathematical relationships existed between the liquid limit, the P.I. and particle size distributions of these soils. He introduced a "soil constant" (t) to represent the particle size distribution, where

$$t = \frac{x}{a} \tag{15}$$

"x" is the per cent by weight of material passing the No. 200 B.S. sieve and "a" is a function of the area under the distribution curve for the coarser particles, specifically

$$a = \frac{\Sigma y}{100n} \tag{16}$$

in which y = the ordinate of the curve at sizes equivalent to the openings in the following set of B.S. sieves: No.'s 200, 100, 52, 25, 14 and 7.
n = the number of ordinates taken (six).

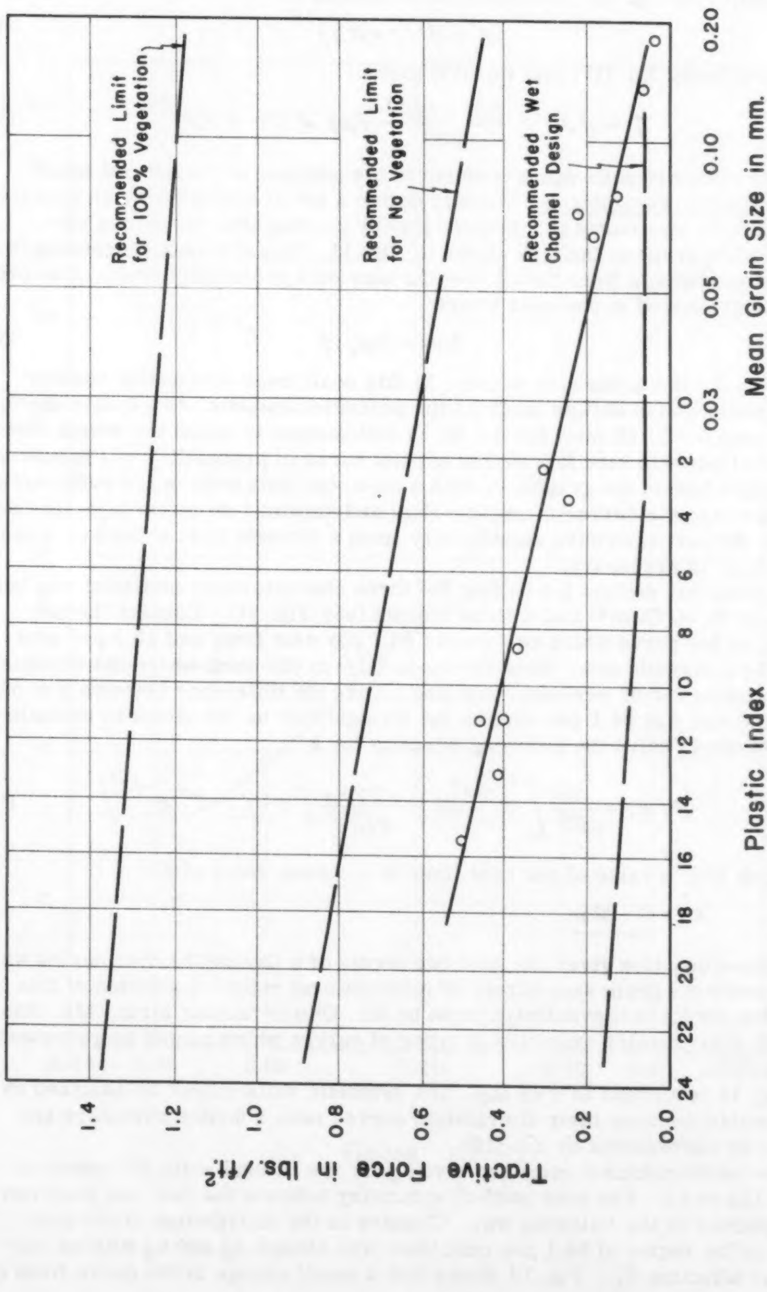

Figure 9 Limiting Bed Tractive Forces for Ephemeral Channels

The statistic "t" has been plotted against θ for the soils in the present investigation in Fig. 10. The equation of the line is

$$\theta^{\circ} = 41(t + 0.2)^{1.8} \tag{17}$$

Substituting Eq. (17) into Eq. (12) gives

$$T_c = 0.02 + \frac{(S_r + 180)}{1000} \tan 41(t + 0.2)^{1.8} \tag{17a}$$

The synthetic soils again conform to the analysis of the natural soils.

Statistical Parameters.—We may define a set of statistics which describe the particle size curve in a precise way by plotting this curve on a phi-probability graph of the type shown in Fig. 11. The abscissa represents the weight percentage finer than a specific size on a probability scale. The ordinate is graduated in phi units where

$$\phi = -\log_2 d \tag{18}$$

in which d = the grain size in mm. In this scale each succeeding number represents a size half as large as the preceding number. (d = 0.25 mm, for $\phi = 2$, and d = 0.125 mm. for $\phi = 3$). A soil sample in which the weight distribution of particle size follows the normal curve of probability will appear as a straight line in the graph. In such a case, two parameters are sufficient to characterize the curve—mean size (M_{ϕ}) and standard deviation (σ_{ϕ}). In cases where the curve deviates significantly from a straight line, a third parameter, skew (k_{ϕ}), is necessary.

A graphical method for finding the three characteristic statistics has been given by G. H. Otto[4] and it is as follows (see Fig. 11): Connect the two points on the curve which represents 84.1 per cent finer and 15.9 per cent finer by a straight line. Read the mean (M_{ϕ}) at the point where the straight line crosses the 50 per cent finer line. Take the difference between ϕ at 50 per cent and ϕ at 84.1 per cent on the straight line as the standard deviation. To find skew, solve the following equation for k :

$$F(\phi) = \frac{100}{\sqrt{2\pi}} \int_0^z e^{-\frac{1}{2}z^2} dz - \frac{100 k_{\phi}}{6\sqrt{2\pi}} \left[1 - (1 - z^2) e^{-\frac{1}{2}z^2} \right] \tag{19}$$

in which $F(\phi)$ = value of per cent finer at a chosen value of ϕ.

$$z = \frac{\phi - M_{\phi}}{\sigma_{\phi}}$$

The above equation gives the first two terms of a Gram-Charlier series which represents the grain size curves of many natural soils. A solution of this equation for k_{ϕ} is conveniently given by Mr. Otto in tabular form. Mr. Otto's article also contains examples of types of curves which cannot be represented by Eq. (19).

Fig. 13 is a graph of θ vs M_{ϕ}. The synthetic soils cannot be analyzed by this method because their distribution curves have a double curvature and cannot be represented by Eq. (19).

The relationships θ vs σ_{ϕ} and θ vs k_{ϕ} for the natural soils are shown in Figs. 12a and b. The near perfect symmetry between the last two plots can be explained in the following way. Changes in the distribution of the sand sizes in the region of 84.1 per cent finer will change σ_{ϕ} and k_{ϕ} without materially affecting T_C. Fig. 14 shows that a small change in the curve from a to

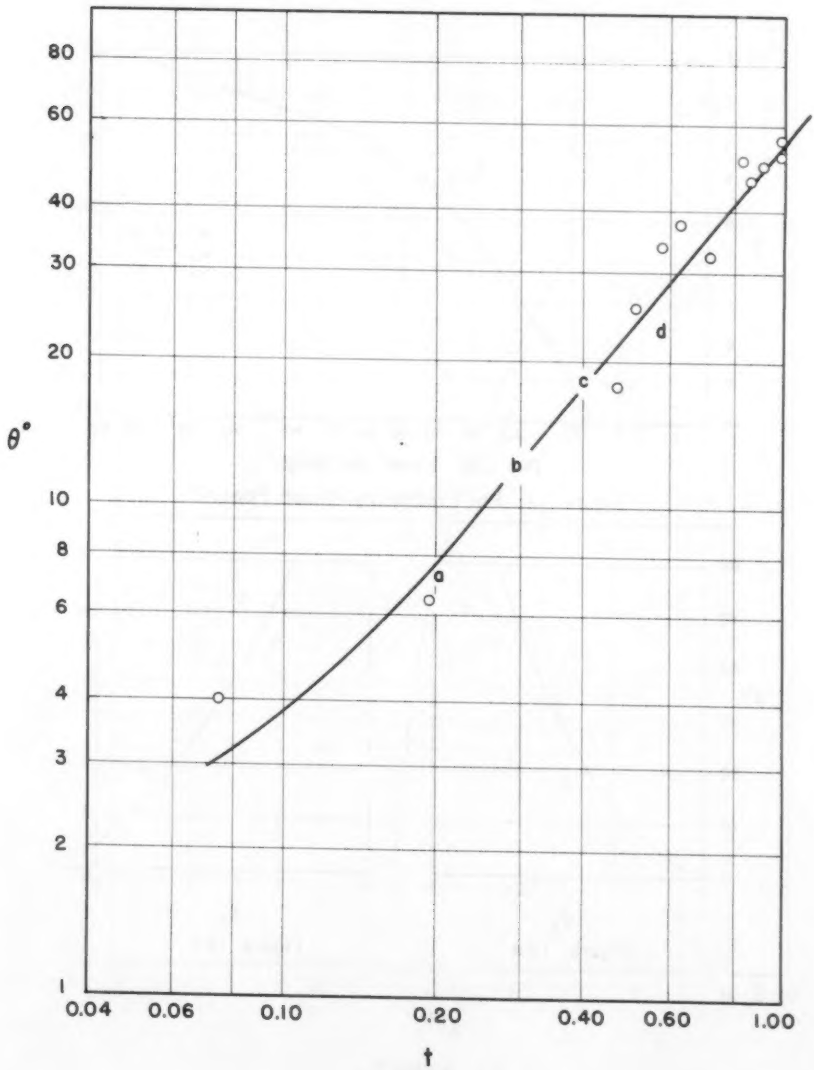

Figure 10

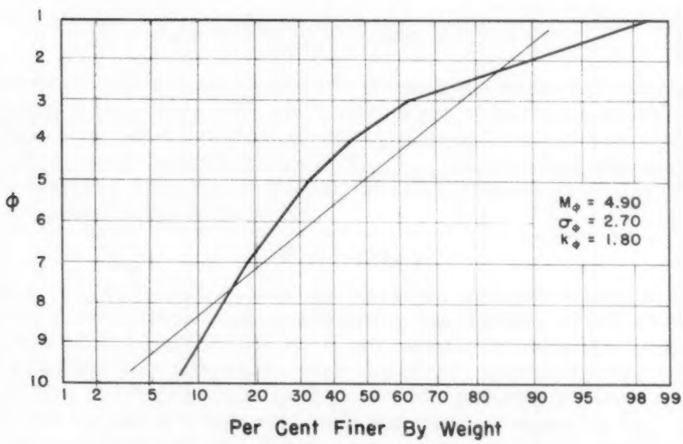

Figure 11 Phi Probability Graph Paper

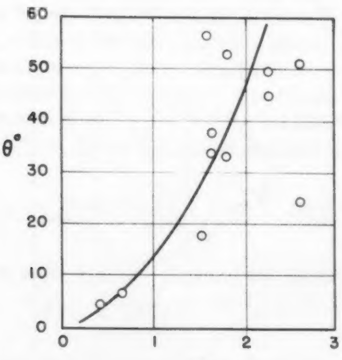

Figure 12a

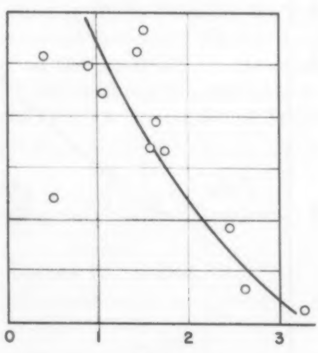

Figure 12b

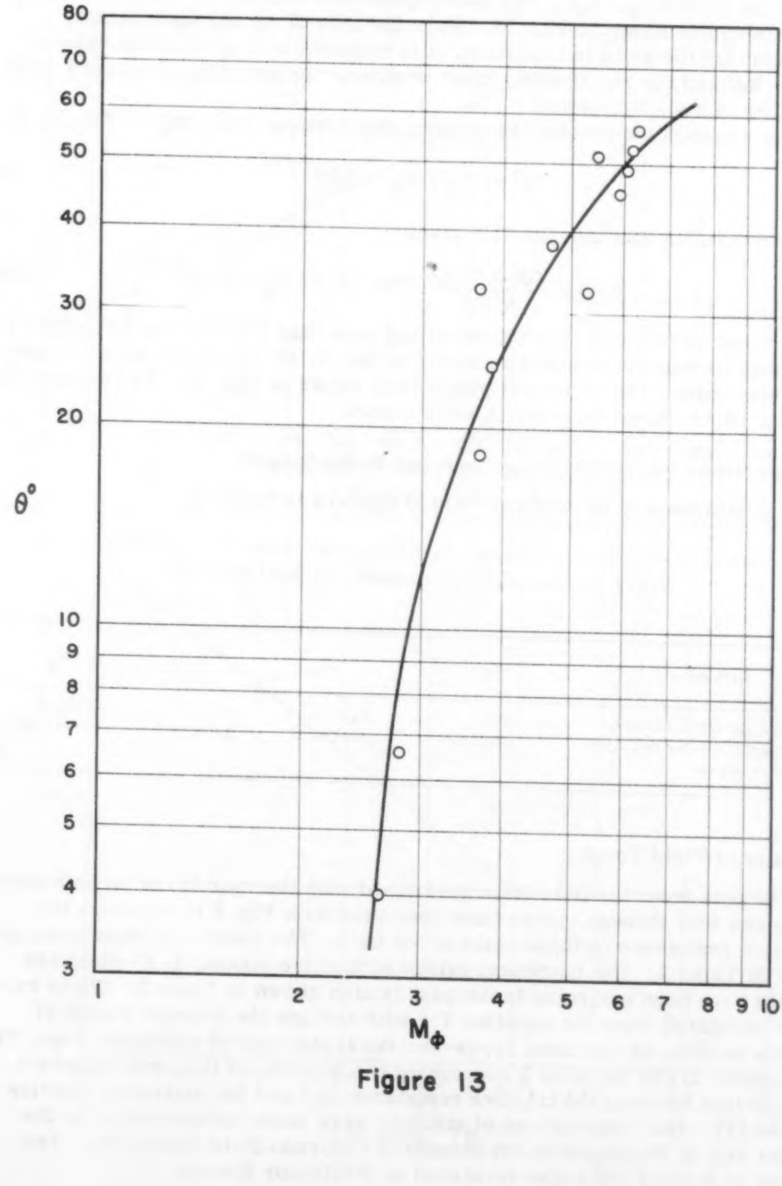

Figure 13

b will change σ_ϕ from 1.30 to 2.00, and k_ϕ from 1.85 to 1.12. σ_ϕ is raised and k_ϕ lowered by about the same amount. To eliminate this effect, terms in the equation for T_c which explain the contribution of σ_ϕ and k_ϕ will probably be of the form $f(\sigma_\phi + k_\phi)$. The interdependence of these two variables for the soils tested is shown in Fig. 15. Since the sum of σ_ϕ and k_ϕ is almost constant for the soils in this study, it is impossible to predict the value of $f(\sigma_\phi + k_\phi)$ and the relationship must remain in terms of M_ϕ alone until other families of soils are tested.

The equation expressing the relationship between θ and M_ϕ in Fig. 13 is

$$\theta^\circ = 15.5\,(M_\phi - 2.3)^{0.905} \tag{20}$$

and substituting this into Eq. (12) gives

$$T_c = 0.02 + \frac{(S_r + 180)}{1000}\tan 15.5\,(M_\phi - 2.3)^{0.905} \tag{20a}$$

This equation is invalid for values of M_ϕ less than 2.3. In this form, this result may be superposed on the results of Mr. E. W. Lane's[5] work for noncohesive soils. The combined results are shown in Fig. 16. The dashed lines in Fig. 16 are taken from Mr. Lane's report.

Comparison of Accuracy of Methods for Estimating θ

A comparison of the methods tested appears in Table 2.

Table 2.--Accuracy of Methods for Estimating θ.

Method	U_f	P.I.	t	M_ϕ
Average Difference Between Observed and Computed θ	10.7 percent	4.7 percent	9.7 percent	11.7 percent

Results of Field Tests

The data from the field tests performed with the vane borer on soil sample locations four through eleven have been used with Fig. 6 to calculate the tractive resistance of these soils in the field. The results of these tests appear in Table 3. The maximum values of tractive stress, T, to which the canals have been subjected in the past is also shown in Table 3. These values were computed from the equation T = wRS and are the average values of stress existing on the canal cross-section at the time of maximum flow. The conclusion drawn in Table 3 concerning the stability of the canal follows a comparison between the tractive resistance (S_h) and the maximum tractive stress (T). The observations of stability were made independently by the writer and by Professor Daryl Simons of Colorado State University. The values of R and S were also furnished by Professor Simons.

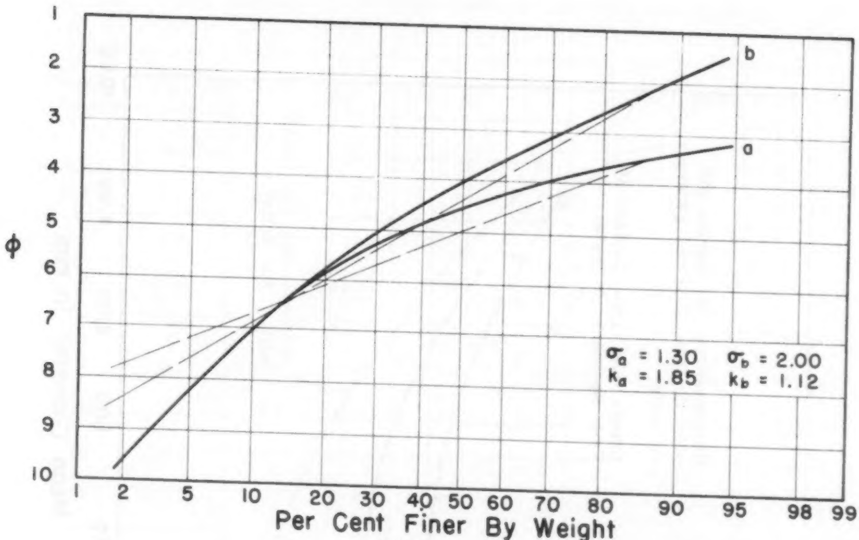

Figure 14

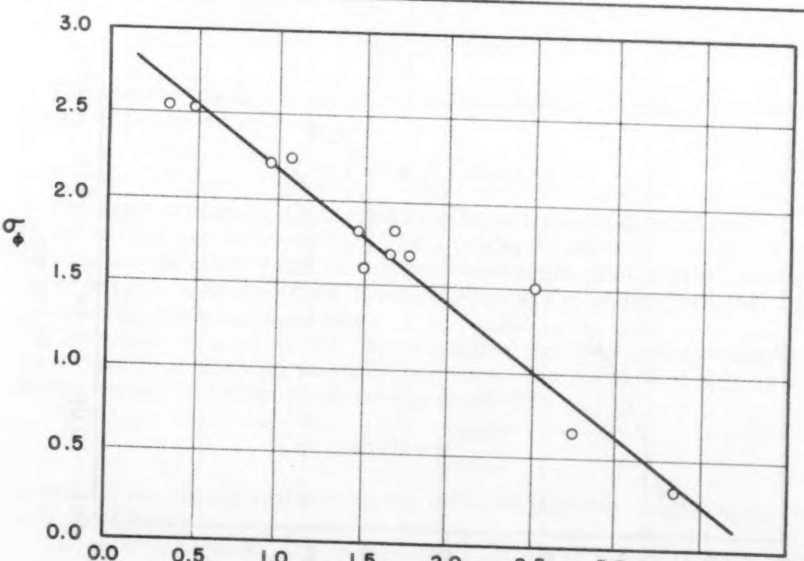

Figure 15

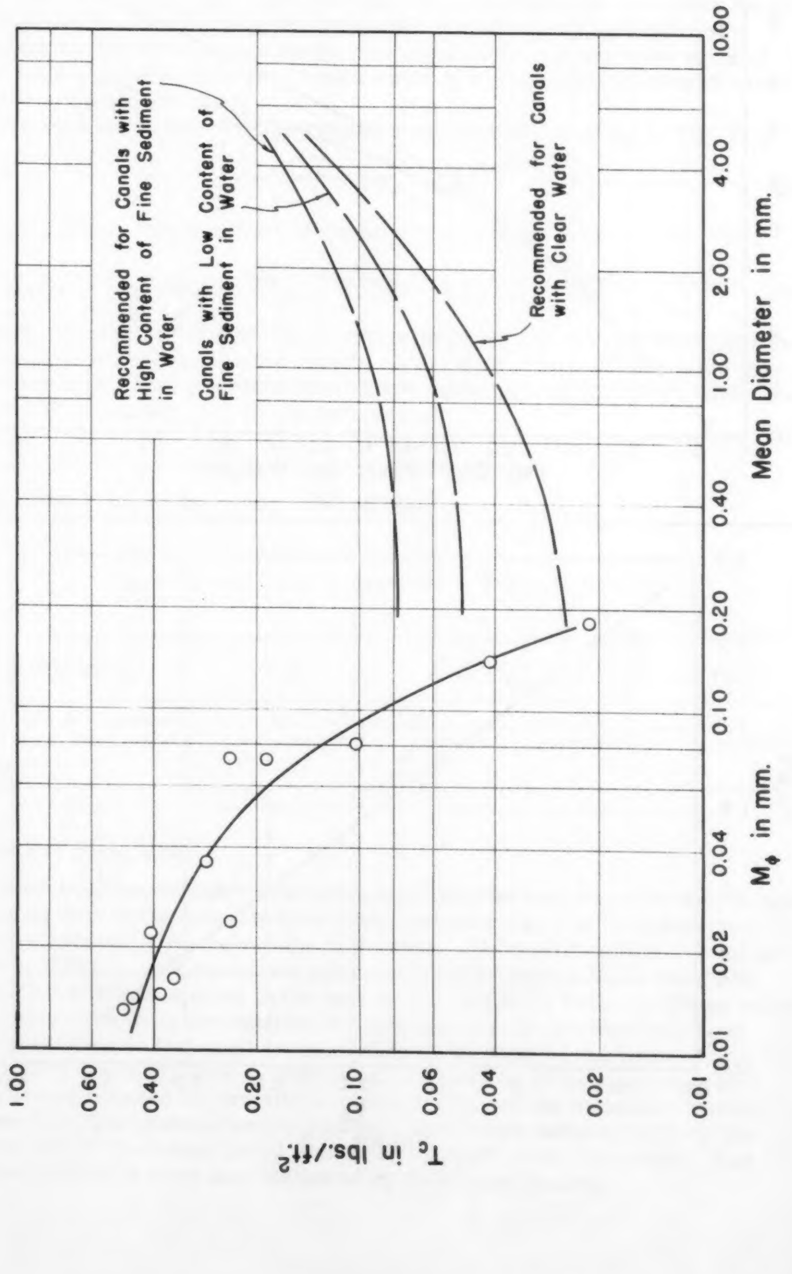
Recommended for Canals with High Content of Fine Sediment in Water
Canals with Low Content of Fine Sediment in Water
Recommended for Canals with Clear Water
Mean Diameter in mm.
M$_\phi$ in mm.
T_c in lbs./ft.^{2}
0.01
0.02
0.04
0.10
0.20
0.40
1.00
2.00
4.00
10.00
0.01
0.02
0.04
0.06
0.10
0.20
0.40
0.60
1.00

Table 3.--Results of Field Tests of Tractive Resistance

Soil Sample	S_v $\frac{\text{lbs.}}{\text{ft.}^2}$	S_h $\frac{\text{lbs.}}{\text{ft.}^2}$	R	S X 10^6	T $\frac{\text{lbs.}}{\text{ft.}^2}$	Conclusion	Observation
4	190	.240	1.82	253	.0288	stable	stable
5	209	.100	1.78	387	.0430	stable	stable
6	178	.270	1.82	294	.0334	stable	stable
7	48	.225	4.66	63	.0184	stable	stable
8	345	.600	4.63	74	.0214	stable	stable
9	238	.270	4.78	130	.0388	stable	stable
10	322	.630	2.58	110	.0178	stable	stable
11	167	.510	3.57	.114	.0254	stable	stable

CONCLUSIONS

The critical tractive stress of the soils tested by the writer may be represented by the following equation:

$$T_c = 0.02 + S_v f(P) + f_1(P) \tag{21}$$

The first term in Eq. (21) is believed to be strength derived from a pressure increase on the surface of the soil in the jet test caused by a momentum change in the water flow in the vertical direction. This strength will not be present in normal field flow conditions and must be subtracted from values of T_c found in the jet test.

The second term in Eq. (21) shows a linear increase in resistance to hydraulic shear as the vane strength increases. The coefficient of S_v is dependent on the soil properties and was found to be

$$f(P) = \frac{\tan\theta}{1000} \tag{22}$$

in which θ may be estimated from the different methods treated in the Discussion of Results.

The third term in Eq. (21) represents a source of hydraulic strength which does not come into play in resisting the shear caused by the vane borer. This term also depends on the soil properties as represented by the angle θ . It was found to be

$$f_1(P) = 0.180 \tan\theta \tag{23}$$

The several methods of presenting the results of this study are each useful in predicting values of tractive resistance for cohesive soils. Some comments concerning the use of these methods follow.

1. The most accurate method of estimating T_c for soils with a P.I. between five and sixteen proved to be the method based on P.I. (see Eq. (14a)).
2. A prediction of T_c based on the per cent of silt and clay in the soil was quite accurate for all of the soils tested. It is apparent, however, that this method is useless to predict differences in T_c for soils which have no sand component, and that it may give inaccurate results for soils with only a small amount of sand (see Eq. (13a)).
3. The methods based on characteristics of the particle size curves are useful for the complete range of soils tested. It is the opinion of the writer that the method based on the phi-probability curve describes the grain size curve very well, and that this method could be developed into a function of all three characteristics—M_ϕ, σ_ϕ, and k_ϕ—if information were available on cohesive soils from several source regions. Correlation of soil properties with grain size gives good results in this investigation because the effects of changes in strength due to differences in chemical and mineralogical content, and due to differences in soil structure, are accounted for by the vane test.

The investigation of cohesive channel beds using the experimental methods described in Experimental Procedure is not limited to the range of soil properties appearing in this study. The same methods could be used with the object of extending the information obtained here to finer and more plastic soils. The same methods could also be used to investigate the critical tractive resistance of beds and banks of ephemeral streams. The strength measured by the vane borer in this case would include apparent cohesion caused by partial saturation. The methods could also be extended to the study of erosion from farm and forest lands which are subjected to water flow during periods of storm.

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Proceedings of the American Society of Civil Engineers

MEASUREMENT OF THE PERMEABILITY OF TRI-AXIALLY ANISOTROPIC SOIL[a]

Marinus Maasland,[1] A. M. ASCE, and Don Kirkham[2]

ABSTRACT

The effect on piezometer measurements of an anisotropy in the horizontal plane is analyzed. Such an anisotropy does not significantly affect the numerical value of the shape factor of the piezometer cavity. It may consequently be ignored in the computation of soil permeability for the type of measurements considered.

SYNOPSIS

The permeability (in this paper designated as hydraulic conductivity) of soil often varies with direction, and a study of the flow of ground water through anisotropic soil is therefore of interest. The hydraulic conductivity may be determined from measurements in a piezometer tube (Hvorslev, 1951; Kirkham, 1955). In the following, the effect on piezometer measurements of an anisotropy in the horizontal plane is considered in detail. It is shown that such an anisotropy does not have a significant effect on the numerical value of the shape factor of the piezometer cavities. This fact permits some simplification in the calculation of the components of hydraulic conductivity from field measurements. The theoretical results presented may also be used to

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evaluate the effect of directional variations in the hydraulic conductivity on the flow of ground water into wells.

INTRODUCTION

The piezometer method of measuring the hydraulic conductivity of soil (of dimensions LT^{-1}) below the water table is described by Luthin and Kirkham (1949), Hvorslev (1951), and Kirkham (1955). It consists, basically, of measuring the rate of rise of the water level in a tube immediately after some water has been pumped out, the tube having been sunk into the soil to a depth below the water table. The tube usually has a small cylindrical cavity below its bottom and is tightly fitted into the soil to prevent leakage along the side. The rate of rise is dependent on the hydraulic conductivity, the radius of the piezometer tube, the shape of the cavity, and the hydraulic head inside and outside the tube (Fig. 1).

Hvorslev (1951), Childs (1952), and Maasland (1957, pp. 266-273, 275-280) show how the piezometer method may be used to evaluate the horizontal and the vertical hydraulic conductivity components of a soil when there is no anisotropy in the horizontal plane. Maasland (1957, pp. 283-284) and Childs (1952) discuss in a general way methods to account for an anisotropy in the horizontal plane in piezometer measurements. Childs (1952) and Childs, et al. (1953), describe a double well method for determination of the horizontal hydraulic conductivity components. Childs, et al. (1957), have used a combination of the double well and piezometer methods to determine the horizontal and vertical hydraulic conductivities of soils, it being assumed that there was no anisotropy in the horizontal plane.

In the following, piezometer measurements in tri-axially anisotropic soil will be considered in detail. The effect of an anisotropy in the horizontal plane will be evaluated quantitatively.

The Piezometer Method in Isotropic Soil

For isotropic conditions, Kirkham (1945) gives a general formula for flow into cavities at the base of piezometers:

$$dQ/dt = \pi r^2\ (dh/dt) = KSh \tag{1}$$

where dQ/dt = rate of inflow into the piezometer (L^3T^{-1})

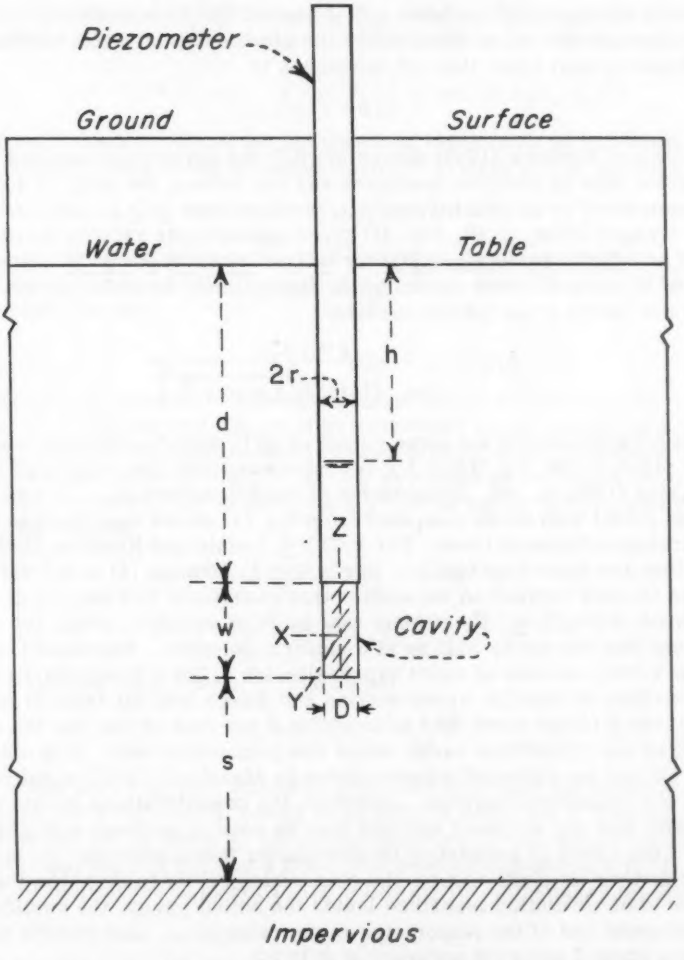
dh/dt = rate of rise of the water level in the piezometer (LT^{-1})

r = inside radius of the piezometer tube (L)

K = hydraulic conductivity (LT^{-1})

S = a constant for a given flow geometry (L)

Hvorslev (1951) uses Q where we use dQ/dt. The factor S, sometimes called a form or shape factor (Hvorslev, 1951; Maasland, 1957, p. 265), may be defined as a factor which, when multiplied by the hydraulic conductivity and by the difference in piezometric head across a flow system, yields the total quantity of flow per unit time entering or leaving the system. For piezometers in homogeneous isotropic soil the shape factor depends upon whether or not



there is an impermeable layer below the homogeneous soil. If there is an impermeable layer, the formula, in functional notation, which gives S, may be written as

$$S/D = f(d/D, w/D, s/D), \tag{2}$$

where (Fig.1)

D = diameter of the cavity

d = depth of the bottom of the tube below the water table

s = depth of the impermeable layer below the bottom of the cavity

w = length of the cavity

If there is no impermeable layer and if further the piezometer cavity is at great depth and the space occupied by the piezometer tube (but not the cavity at its base) is neglected, then (2) simplifies to

$$S/D = f(w/D). \tag{3}$$

Luthin and Kirkham (1949) determine S/D for cylindrical cavities below a piezometer tube by electric analogues and list values, for $w/D = 0$ to $w/D = 8$, for piezometers in an infinite medium, in which case S/D is independent of d and s. Zangar (1953, p. 50, Fig. 41) gives approximate values, also for the case of an infinite medium, of S/D for $w/D = 1/2$ to $w/D = 2000$. Hvorslev (1951, p. 31, case 8) gives the following approximate formula for flow into a cylindrical cavity in an infinite medium

$$S/D = \frac{2\pi(w/D)}{\log_e\left[(w/D) + \sqrt{1 + (w/D)^2}\right]} \tag{4}$$

and this formula agrees for large values of w/D, with the formula used by Zangar (1953, p. 68, Eq. (15a)) for his values and with the simplified formula of Hvorslev (1951, p. 30). Comparison of the S/D values of Luthin and Kirkham (1949) with those computed from Eq. (4) shows that there is reasonable agreement between them. For $w/D > 4$, Luthin and Kirkham (1949) report values that are somewhat higher. For $w/D \leq 1$, formula (4) is not valid. Formula (4) was derived on the assumption of uniform flux density along a line source of length w. No attempt was made to satisfy in detail the requirement that the cavity wall be at a uniform potential. Maasland (1957, p. 272, Eq. V.20c) derives an exact expression for S/D for flow towards ellipsoidal cavities of circular cross section and shows (see his table 6) that for $w/D > 3$, the S factor given by 4 is to within 3 per cent of that for the inscribed ellipsoid of the cylindrical cavity below the piezometer tube. It is noted that neither (4) nor the ellipsoid solution given by Maasland (1957) is entirely correct for cylindrical cavities. However, the considerations in this paragraph show that the ellipsoid solution may be used to evaluate with sufficient accuracy the effect of anisotropy on piezometer measurements. In fact, to proceed further we shall assume the piezometer cavity to be of the form of an ellipsoid of revolution inscribed inside the actual cavity, the vertical axes of the ellipsoid and of the piezometer cavity coinciding. Our results should be good to about 3 per cent accuracy if $w/D > 3$.

For the convenience of the reader, the major results of the theory of anisotropy will be repeated before continuing the analysis of the ellipsoid problem. These results are included even though comprehensive discussions of the theory are available elsewhere.

Anisotropy Theory Applied to Piezometer Measurements

The theory of fluid flow through anisotropic media is given by Vreedenburgh (1936) and Muskat (1937); it has recently been discussed in detail by Ferrandon (1948), Scheidegger (1957, pp. 63-66), and Maasland (1957, esp. pp. 218-223). From this theory, it follows that an anisotropic medium may be transformed into an apparent isotropic one by shrinkage or expansion of the coordinates of the flow system. Let K_x, K_y, and K_z be the hydraulic conductivities in the principal directions x, y, and z of anisotropy, and let K_o by an arbitrary

constant of the same dimensions as K_x, K_y, and K_z. The coordinates of the transformed system x', y', and z' are then defined by the following formulas

$$x' = (K_o/K_x)^{\frac{1}{2}} x, \quad y' = (K_o/K_y)^{\frac{1}{2}} y, \quad \text{and } z' = (K_o/K_z)^{\frac{1}{2}} z \tag{5}$$

The hydraulic conductivity of the transformed medium K_t is related to the hydraulic conductivities of the actual anisotropic system by the equation

$$K_t = (K_x K_y K_z / K_o)^{\frac{1}{2}} \tag{6}$$

In the following we will assume that the principal directions x and y are in the horizontal plane, and that $K_x \neq K_y \neq K_z$. Also, with Maasland (1957, p. 283) we shall choose

$$K_o = (K_x K_y)^{\frac{1}{2}} \tag{7}$$

The origin of coordinates will be taken at the center of a piezometer cavity.

From (5) it follows that the circular horizontal cross-section of a cylindrical cavity below a piezometer tube becomes elliptic in the transformed flow system. We find from (5) and (7), for the semiaxes a_x and a_y of the horizontal elliptic cross-section and for the length w' of the cavity in the transformed system, the results

$$\begin{aligned} a_x &= \tfrac{1}{2}(K_y/K_x)^{\frac{1}{4}} D, \quad a_y = \tfrac{1}{2}(K_x/K_y)^{\frac{1}{4}} D, \\ \text{and } w' &= \left[(K_x K_y)^{\frac{1}{2}} / K_z\right]^{\frac{1}{2}} w \end{aligned} \tag{8}$$

For K_t we obtain from (6) and (7) the result

$$K_t = \left[(K_x K_y)^{\frac{1}{2}} K_z\right]^{\frac{1}{2}} \tag{9}$$

The shape factor S_a (the subscript a is used to denote the condition of anisotropy) for a cylindrical cavity in an infinite anisotropic medium is dependent on the cavity depth w, the cavity diameter D, and the hydraulic conductivities K_x, K_y and K_z; and a functional relationship similar to (3) may, in view of (8), be written in either of the following two dimensionless forms

$$S_a/D = F(a_x/D, a_y/D, w'/D) \tag{10}$$

or

$$S_a/D = G(K_x/K_y, w'/D) \tag{10a}$$

which value is identical to the factor S for a particular cylindrical cavity with elliptical cross-section in an infinite isotropic medium; the values of a_x, a_y, and w' are defined by (8). Since the physical dimensions of the cavity (now a right elliptic cylinder) in the equivalent isotropic system are now known, the factor S_a/D may be obtained from such analogue measurements as were made by Luthin and Kirkham (1949). The electrodes needed must have the elliptical cross-sections, and the S values would need to be determined for several cavity lengths and for cross-sections of varying degrees of ellipticity. A few analogue tests have been made by Maasland (1957, p. 284), the work requiring, as Childs (1952, p. 533) has noted, considerable labor. Rather than to continue with electric analogues it is possible to approach the problem in a different manner. Progress can be made by considering, as was justified below Eq. (4),

ellipsoidal instead of cylindrical cavities and attacking the problem analytically, which we proceed to do.

The Factor S for an Ellipsoidal Cavity in Isotropic Soil

For the case that $K_x \neq K_y \neq K_z$, we need to know the S factor for an ellipsoid with three different major axes, which problem may be solved by using some results from the theory of electricity similar to those used by Evans and Kirkham (1950) and Maasland and Kirkham (1955). It follows that the constant S is given by

$$S = 4\pi C \tag{11}$$

C being the electrostatic capacity of an ellipsoid or disk in an infinite medium.

Smythe (1939, p. 111, Eq. (4)) shows, for an ellipsoid with semiaxes a, b, and c in the principal directions x, y, and z, that

$$1/C = \frac{1}{2}\int_{0}^{\infty} \frac{d\theta}{\sqrt{(a^2+\theta)(b^2+\theta)(c^2+\theta)}} \tag{12}$$

where θ is a variable of integration.

There is a difficulty with (12). The right-hand side is not in tractable or tabulated form. If we assume that $0 < a \leq b < c$, it may be shown, by making the successive changes of variables $\theta = u^2 - a^2$, $u = 1/v$, $v = (c^2 - a^2)^{-1/2}w$, $w = \tan \phi$, that (12) is equivalent to

$$1/C = \left[1/(c^2 - a^2)^{\frac{1}{2}}\right]\int_{0}^{\sin^{-1}p} (1 - k^2 \sin^2 \phi)^{-\frac{1}{2}} d\phi \tag{13}$$

in which

$$k^2 = 1 - (b^2 - a^2)/(c^2 - a^2) \tag{14}$$

and

$$p = \left[(c^2 - a^2)/c^2\right]^{\frac{1}{2}} \tag{15}$$

Formula (13) contains an incomplete elliptic integral of the first kind which can be evaluated from tables.

The Factor S_a for an Ellipsoidal Cavity in Anisotropic Soil

Assume now that $K_x \geq K_y$. It is then found from (8) that

$$a = \tfrac{1}{2}(K_y/K_x)^{\frac{1}{4}}D, \quad b = \tfrac{1}{2}(K_x/K_y)^{\frac{1}{4}}D, \text{ and}$$
$$c = \tfrac{1}{2}\left[(K_x K_y)^{\frac{1}{2}}/K_z\right]^{\frac{1}{2}} w = \tfrac{1}{2}w' \tag{16}$$

It follows from the condition stated in the previous paragraph that (13) is valid only if c is larger than a or b.

From (15) and (16) we obtain

$$p = \left\{ \frac{(w'/D)^2 - (K_y/K_x)^{\frac{1}{2}}}{(w'/D)^2} \right\}^{\frac{1}{2}} \tag{17}$$

and from (14) and (16)

$$k^2 = 1 - \frac{(K_x/K_y)^{\frac{1}{2}} - (K_y/K_x)^{\frac{1}{2}}}{(w'/D)^2 - (K_y/K_x)^{\frac{1}{2}}} \tag{18}$$

We finally find from (11), (13) and (16) the result

$$S_a/D = 2\pi \left[(w'/D)^2 - (K_y/K_x)^{\frac{1}{2}} \right]^{\frac{1}{2}} \frac{1}{\int_0^{\sin^{-1} p} (1 - k^2 \sin^2 \phi)^{-\frac{1}{2}} d\phi} \tag{19}$$

In this formula, p and k^2 are given by (17) and (18). It is easy to show that, for $K_x = K_y$ (isotropy in the horizontal plane) and $w'/D > 1$ with k^2 in (19) now equal to unity, (19) leads to Eq. (V.20c) of Maasland (1957, p. 272) mentioned before.

Using the tables for the incomplete elliptic integral of the first kind, as listed by Byrd and Friedman (1954), S_a/D has been calculated from (19) for some values of w'/D and K_x/K_y; the results are given in Table 1. S_a/D is given for values of K_x/K_y as high as 25. So far, such high values of K_x/K_y have not been encountered in the field. In fact, actual measurements indicate values that are much lower (1.4) (Scheidegger, 1954; Johnson and Breston, 1951; and Johnson and Hughes, 1948). For $K_x/K_y = 1$, S_a/D may be calculated from (19) or from (V.20c) of Maasland (1957).

Table 1: Values of S_a/D for various values of w'/D and (K_x/K_y).

w'/D	$K_x/K_y = 1$	$K_x/K_y = 2.25$	$K_x/K_y = 4$	$K_x/K_y = 9$	$K_x/K_y = 16$	$K_x/K_y = 25$
4	11.79	11.89	12.12	12.52	12.96	13.37
10	20.91	21.01	21.28	21.89	22.48	23.03
20	34.02	34.16	34.56	35.38	36.13	36.90
40	57.32	57.54	58.12	59.04	60.28	61.44

Discussion

Since Table 1 shows, for all values of w'/D, only a small increase in S_a/D with increasing K_x/K_y, it appears justified to conclude that for practical purposes it is sufficient to use the S_a/D value for $K_x/K_y = 1$ for all values of K_x/K_y. It thus follows that we may restrict our attention to S_a/D values for cavities with a circular horizontal cross-section even if there is an anisotropy

in the horizontal plane. Childs, et al. (1957, p. 31), assume in their field measurements of anisotropy that there is no anisotropy in the horizontal plane. This assumption is needed, in particular, for their Eq. (6) which applies to flow into a partially penetrating well. Our Table 1 now shows that this assumption is not necessary if the value of $(K_x/K_y)^{1/2}$ is known. The assumption is necessary if measurements with the double well method (Childs, 1952) in a single direction are taken to represent the horizontal component of hydraulic conductivity.

The foregoing results were derived for ellipsoidal cavities. Maasland (1957, p. 283) has derived similar results for flow to or from a circular opening in a horizontal impervious layer. Since an anisotropy in the horizontal plane results in a distortion of the flow configuration in the radial direction, it would appear that conclusions drawn from a study of flow towards ellipsoidal cavities with circular cross-sections are also applicable to cylindrical cavities with circular cross-sections.

A numerical example may serve to clarify further the implications of the theoretical results. In the example it will be assumed, as is done by Hvorslev (1951) and others, mentioned above (4), that the shape factor for the inscribed ellipsoid of a cylindrical cavity may be used for the cylindrical cavity itself. Let a piezometer be so located that the effects of s and d (Fig. 1) may be neglected while we have $w/D = 4$. Furthermore, let the hydraulic conductivities in the principal directions of anisotropy be $K_x = 4$, $K_y = 1$, and K_z (vertical) = 0.08 foot per day. Since $\left[(K_x/K_y)^{1/2} / K_z\right]^{1/2} = 5$ in this problem, we find from (16) that $w'/D = 5 \times 4 = 20$. It follows that a shape factor is needed for $w'/D = 20$ and $K_x/K_y = 4$ for which a value is listed in Table 1. We find that $S_a/D = 34.56$. If the anisotropy in the horizontal plane had been ignored and the value $K_h = (K_x/K_y)^{1/2} = 2$ had been used as a constant substitute value for the horizontal hydraulic conductivity, we would have found for $w'/D = 20$ and now with $K_x/K_y = 1$ that $S_a/D = 34.02$ (Table 1). The value 34.02 differs from that for $K_x/K_y = 4$ by only 1.5 per cent.

It is of interest to note that the methods for separating the horizontal and vertical hydraulic conductivities from a combination of piezometer measurements, as described by Maasland (1957, p. 280), will essentially yield $(K_x/K_y)^{1/2}$ for the horizontal hydraulic conductivity component, (K_h in the notation of Maasland, 1957, p. 280), and K_z for the conductivity component in the vertical direction.

Smythe (1939, pp. 110 and 111) gives formulas for the potential distribution around electrically charged ellipsoids. Those formulas may be used to calculate the hydraulic head distribution around piezometers in soil having an anisotropy in the horizontal plane or in all three directions.

The results of this paper may be applied to flow towards partially penetrating wells in anisotropic media since a well cavity may be considered as a piezometer cavity with a large value of the ratio w/D. It is finally noted that an anisotropy in the horizontal plane may significantly affect desirable spacings of wells in multiple-well systems if the hydraulic conductivity shows a consistent directional trend over a relatively wide area.

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ANALYSIS OF CONCRETE SLABS ON GROUND

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ABSTRACT

A new theory is developed for calculating the stresses and deflections in concrete slabs of finite size supported on ground. The theory accounts for weight, superimposed loads, warping due to temperature and moisture gradients, and for the condition that the slab may be only partially supported by the ground.

The validity of the new theory is assessed in the light of existing theories and available measured data.

INTRODUCTION

Inadequacies in the performance of structural slabs on ground for buildings and homes, or of concrete pavements for highways and airports, have been the cause of increasing concern among civil engineers interested in these problems. It has long been known that such slabs are subject to considerable warping because differential length changes occur when temperature and moisture gradients develop in the slab. Early attempts to account for warping effects were semi-empirical in nature and met with limited success. With the advent of rational analyses, design concepts improved substantially; however, in recent years increasing recognition has been accorded the fact that the usefulness of these analytical tools is seriously restricted by the common assumption that the slab maintains full contact with its support at all times.

As early as 1910, the Office of Public Roads(4) conducted field and laboratory tests to study the effects of expansion and contraction of concrete pavements. In 1922, tests conducted on the Bates Experimental Road led to the discovery that temperature differences between upper and lower pavement

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surfaces result in considerable warping of the slab. By 1924, Older(11) had developed a semi-empirical formula for the required slab thickness on the assumption that the corner was entirely unsupported.[3]

The first completely rational theoretical analysis of the structural action of concrete slabs on ground was contributed by H. M. Westergaard(20) in 1926. In 1927, Westergaard(21) extended his analysis to the consideration of the stresses and deflections induced in the slab by temperature gradients. The generality and elegance of Westergaard's classic analyses led to their early adoption and widespread use as design criteria. With modifications that account for the effect of adjacent loads,(15) of impact(1) and of load repetitions,(12,16) Westergaard's approach still provides the most reliable basis for design.

Recent experimental studies and field observations(3) have cast considerable doubt on the validity of Westergaard's assumption that the subgrade provides complete support to the pavement at all times.[4] In 1944, the California Division of Highways began intensive field and laboratory studies to determine the cause of distress at the joints of concrete pavements. Reporting the results of this study in 1951, Hveem(6) commented as follows: "Engineers have paid much attention to subgrade support for concrete pavements at or near the planned joints in the pavement, but if it is recognized that for a considerable portion of time rigid pavements do not rest on the subgrade for several feet either side of the joint, it seems pertinent to ask whether Westergaard's K is a significant index for the design of such pavements."

A survey of the performance of industrial floors on ground made by the Missouri River Division, Corps of Engineers, in 1955,(2) noted voids under slabs as much as 3/8" deep, extending 2 to 5 feet away from the joint. It was observed that the high points occurred at the corners of these slabs. As the slabs were inside buildings and not subject to large changes in temperature, it was reasoned that the warping was primarily the result of moisture differences between slab surfaces. Geldmacher, et al,(3) in 1957, determined experimentally that the load bearing characteristics of a highway pavement varied diurnally with environmental changes. Direct evidence of voids beneath pavements may be found in a recent paper by Hveem,(7) in which a view through a core hole in a pavement slab shows approximately 1/4" clearance between the slab bottom and the subbase. The slab also shows a crack in this area.

From the evidence cited, it may be concluded that temperature and moisture gradients that develop in concrete slabs supported on ground can, under prevalent field conditions, cause sufficient warping of the slab to result in partial support. The weight of the slab, and of superimposed loads, produce more critical stress conditions in partially supported slabs than those

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3. A "corner formula" to account for weak subgrade conditions was originally proposed by A. T. Goldbeck (Public Roads, Vol. 1, April 1919, p. 37). Older's corner formula was subsequently refined by Westergaard,(22) Kelley,(8) Spangler,(14) Bradbury(1) and Pickett.(12) These latter formula account for the lack of subgrade support due to warping effects in a semi-empirical manner.
4. As reported by Sutherland,(17) after seeing the temperature data obtained in the Arlington, Va., investigations,(18) Westergaard expressed the opinion that his analysis might not be applicable for temperature differentials of such magnitudes (4° F per inch of slab thickness when the top of the slab was warmer than the bottom).

which would be computed on the basis of full subgrade support. It is the authors' opinion that this fact is responsible for some of the principal discrepancies between anticipated and observed performance of concrete slabs on ground. It is also their belief that a major portion of the data relating to measured strains and deflections in concrete slabs on ground are incapable of interpretation, as strain gauges have been applied without due cognizance of warping stresses already extant, and deflections have been measured without knowledge of the support conditions.

In this paper general equations for the deflections and stresses in warped slabs are developed for finite slab sizes, allowing for the condition that warping may result in a partially supported slab. Particular solutions are presented for free edge boundary conditions that account for warping due to temperature and moisture gradients, for the weight of the slab or other uniformly distributed loads, for superimposed peripheral loads, and for concentrated loads at the center. As an example, numerical solutions are obtained for the case of an unloaded slab (apart from its weight), and the results presented in the form of nomographs. Finally, comparisons are made with available measured data and the validity of the new theory is assessed.

Notation

w = deflection, positive in downward direction

q = uniformly distributed load

k = modulus of subgrade reaction

p = reaction of subgrade

h = slab thickness

μ = Poisson's ratio

E = Young's modulus

D = flexural rigidity of the slab = $\dfrac{Eh^3}{12\,(1-\mu^2)}$

ℓ = radius of relative stiffness = $\sqrt[4]{D/k}$

α = linear coefficient of thermal expansion

T = equivalent temperature difference between upper and lower slab surfaces; length changes caused by moisture differences are transformed into equivalent temperature differences.

r = radial distance

a = slab radius

b = radial distance to point of zero deflection

β = $\dfrac{b}{\ell}$

ρ = $\dfrac{r}{\ell}$

$w'(r)$ = slope at point r

$$\nabla_r^2 = \left(\frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr}\right)$$

$$\nabla_r^4 = \left(\frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr}\right)\nabla_r^2$$

$$V(r)^* = -D\left[\frac{d}{dr}\nabla_r^2 w\right]$$ = shear at point r

$$M(r)^* = -D\left[\frac{d^2w}{dr^2} + \frac{\mu}{r}\frac{dw}{dr} + \alpha(1+\mu)\frac{T}{h}\right]$$ = radial bending moment at point r

$$\sigma(r)^* = -\frac{Eh}{2(1-\mu^2)}\left[\frac{M(r)}{D}\right]$$ = normal radial stress at point r, positive denotes tension

σ_o = normal (radial) stress at center of slab

C_j = coefficients

$Z_i(p)$ = Bessel functions

$Z_i'(p)$ = first derivative of $Z_i(p)$

General Considerations

The following analyses consider the bending of circular slabs of uniform thickness resting on homogeneous foundations whose reaction against the slab is a function of the deflection. The loads and subgrade reaction act normal to the slab surfaces and are symmetrical with respect to the center of the slab. The slab is subject to changes in temperature (and/or moisture) which vary linearly with depth but remain constant on all planes parallel to the surfaces of the slab. Assuming that deflections are small in comparison with the thickness of the slab, the following differential equation must be satisfied:[13]

$$D\nabla_r^4 w = q - p \tag{1}$$

To solve Eq. (1), it is necessary that the relationship between the subgrade reaction (p) and the deflection (w) be known. It will be assumed[5] that the subgrade acts as a dense liquid of unit weight k, thus

$$p = k\,w \tag{2}$$

and Eq. (1) becomes

$$D\nabla_r^4 w = q - kw \tag{3}$$

*c.f. Melan, E., and Parkus, H., "Warmespannungen," Springer-Verlag, Vienna (1953).

5. This assumption was first introduced by Winkler in 1867 and formed the basis of Westergaard's classical work.

Eq. (3) was solved by Schleicher[13] in 1926. By changing the dependent variable, $w = \bar{w} + \frac{q}{k}$ and defining the radius of relative stiffness ℓ, as

$$\ell^4 = \frac{D}{k} \tag{4}$$

Eq. (3) is transformed into a homogeneous differential equation of fourth order:

$$\nabla_\rho^4 \bar{w} + \bar{w} = 0 \tag{5}$$

where $\rho = \frac{r}{\ell}$.

Introducing $\rho = \sqrt[4]{-1}\, x = \pm\sqrt{\pm i}\, x$, Eq. (5) becomes

$$\nabla_x^4 \bar{w} - \bar{w} = 0 \tag{6}$$

which in turn can be resolved into

$$\left.\begin{aligned} &\nabla_x^2(\nabla_x^2 \bar{w} + \bar{w}) - (\nabla_x^2 \bar{w} + \bar{w}) = 0 \\ \text{or: } &\nabla_x^2(\nabla_x^2 \bar{w} - \bar{w}) + (\nabla_x^2 \bar{w} - \bar{w}) = 0 \end{aligned}\right\} \tag{7}$$

Therefore, the solution of Eq. (6) is the sum of the solutions of the following two differential equations:

$$\left.\begin{aligned} &\nabla_x^2 \bar{w} + \bar{w} = 0 \\ \text{and } &\nabla_x^2 \bar{w} - \bar{w} = 0 \end{aligned}\right\} \tag{8}$$

The first of Eqs. (8) is Bessel's equation of zero parameter; the second can be transformed into the same form by changing the variable x to $i\psi$. The solution of Eq. (6) thus becomes

$$\bar{w} = A_1 J_0(x) + A_2 J_0(ix) + A_3 Y_0(x) + A_4 Y_0(ix) \tag{9}$$

Where $J_0(x)$ and $Y_0(x)$ are Bessel functions of the first and second kind, respectively, both of zero order. For computational purposes it is more convenient to express Eq. (9) in real functions of the argument ρ. To do this, Schleicher defined the following Z functions:

$$\left.\begin{aligned} Z_1(\rho) &= \frac{1}{2}\left[J_0(\rho\sqrt{+i}) + J_0(\rho\sqrt{-i})\right] \\ Z_2(\rho) &= -\frac{i}{2}\left[J_0(\rho\sqrt{+i}) - J_0(\rho\sqrt{-i})\right] \\ Z_3(\rho) &= Z_1(\rho) + \frac{i}{2}\left[Y_0(\rho\sqrt{+i}) - Y_0(\rho\sqrt{-i})\right] \\ Z_4(\rho) &= Z_2(\rho) + \frac{1}{2}\left[Y_0(\rho\sqrt{+i}) + Y_0(\rho\sqrt{-i})\right] \end{aligned}\right\} \tag{10}$$

Values for these Z functions, their derivatives and assymptotic values, were given by Schleicher. They are more commonly expressed and tabulated in the following manner:

$$Z_1(\rho) = ber(\rho)$$
$$Z_2(\rho) = -bei(\rho)$$
$$Z_3(\rho) = -\frac{2}{\pi} kei(\rho)$$
$$Z_4(\rho) = -\frac{2}{\pi} ker(\rho)$$

The solution of Eq. (3) then takes the form

$$w = \frac{q}{k}\left[1 + C_1 Z_1(\rho) + C_2 Z_2(\rho) + C_3 Z_3(\rho) + C_4 Z_4(\rho)\right] \tag{11}$$

for the case where the slab is fully supported by the subgrade and subject to a uniformly distributed load q.

For a slab entirely unsupported by the subgrade, k = 0, and Eq. (3) reduces to

$$D \nabla_r^4 w = q \tag{12}$$

Eq. (12) is a fourth order linear differential equation with variable coefficients of the Euler-Cauchy type and has for its general solution.[19]

$$w = C_5 + C_6 \ln r + C_7 r^2 + C_8 r^2 \ln r + q \frac{r^4}{64D} \tag{13}$$

Eqs. (11) and (13) are general solutions for the deflections of circular slabs (subject to uniformly distributed loads) for the cases where the slabs are fully supported and entirely unsupported by the subgrade. The C coefficients in Eqs. (11) and (13) are constants of integration and are dependent on the boundary conditions imposed on the slabs.

Partially Supported Slabs

Fig. 1 shows a diametric section through a circular slab whose edges have warped upwards sufficiently to result in a partially supported slab. This upward warping is considered to result from temperature and/or moisture variations that increase linearly with depth. (Variations not linear with depth can be handled similarly. The linear case is treated here because indications are[18] that this case produces the largest stresses in the slab.) The slab is divided into two zones with the inner portion (Zone 1) being a uniformly loaded slab resting on the foundation while the outer region (Zone 2) is unsupported by the foundation.

Thus, for partially supported slabs Eq. (11) is a general expression for the deflections for $r \leq b$ (the point of zero deflection), and Eq. (13) is the corresponding expression for $r \geq b$. From the relations between stress, moment and deflection (see notation, p. 37) the general equations for the normal (radial) stress in the supported and unsupported portions of the slab are, respectively:

$b \geq r > 0$

$$\sigma(r) = \frac{E h}{2(1-\mu^2)}\left[\frac{q}{k\ell^2}\left\{C_1\left[Z_2(\rho) - \frac{(1-\mu)Z_1'(\rho)}{\rho}\right] - C_2\left[Z_1(\rho) + \frac{(1-\mu)Z_2'(\rho)}{\rho}\right] + \right.\right.$$
$$\left.\left. + C_3\left[Z_4(\rho) - \frac{(1-\mu)Z_3'(\rho)}{\rho}\right] - C_4\left[Z_3(\rho) + \frac{(1-\mu)Z_4'(\rho)}{\rho}\right]\right\} + \alpha(1+\mu)\frac{T}{h}\right] \tag{14}$$

$a \geq r \geq b$

$$\sigma(r) = \frac{E h}{2(1-\mu^2)}\left[\frac{C_6}{r^2}(\mu-1) + 2C_7(1+\mu) + C_8(2\mu \ln r + 2\ln r + 3 + \mu) + \right.$$
$$\left. + \frac{q r^2}{16 D}(3+\mu) + \alpha(1+\mu)\frac{T}{h}\right] \tag{15}$$

Nine conditions must be known (eight C coefficients and the value of b) in order to obtain unique solutions for the stress and deflection patterns.

Along the circle of radius b the deflection is zero and conditions of continuity require that the deflection, slope, bending moment, and shear of the inner and outer portions be equal. Also, if the loading and reactive forces are symmetrically distributed about the center of the plate, the slope at the center must be zero. Thus, six conditions are specified:

$$\left.\begin{array}{ll} a) & w_1(b) = 0 \\ b) & w_2(b) = 0 \\ c) & w_1'(b) = w_2'(b) \\ d) & M_1(b) = M_2(b) \\ e) & V_1(b) = V_2(b) \\ f) & w_1'(0) = 0 \end{array}\right\} \tag{16}$$

The subscripts 1 and 2 refer to the supported and unsupported zones of the plate, respectively. Applying the conditions of Eqs. (16) to Eqs. (11) and (13) gives:

$$\left.\begin{array}{l}
C_4 = 0 \\
C_1 Z_1(\beta) + C_2 Z_2(\beta) + C_3 Z_3(\beta) + 1 = 0 \\
C_5 + C_6 \ln b + C_7 b^2 + C_8 b^2 \ln b + \dfrac{q b^4}{64 D} = 0 \\
\dfrac{C_6}{b} + 2C_7 b + C_8 b(2\ln b + 1) + \dfrac{q b^3}{16 D} - \dfrac{q}{k\ell}\left[C_1 Z_1'(\beta) + C_2 Z_2'(\beta) + C_3 Z_3'(\beta)\right] = 0 \\
\dfrac{C_6}{b^2}(\mu-1) + 2C_7(1+\mu) + C_8\left[2 \ln b(1+\mu) + (3+\mu)\right] + \dfrac{q b^2(3+\mu)}{16 D} \\
\quad - \dfrac{q}{k\ell^2}\left\{C_1\left[Z_2(\beta) - \dfrac{(1-\mu)Z_1'(\beta)}{\beta}\right] - C_2\left[Z_1(\beta) + \dfrac{(1-\mu)Z_2'(\beta)}{\beta}\right] + C_3\left[Z_4(\beta) - \dfrac{(1-\mu)Z_3'(\beta)}{\beta}\right]\right\} = 0 \\
\dfrac{q}{k\ell^3}\left\{C_1 Z_2'(\beta) - C_2 Z_1'(\beta) + C_3 Z_4'(\beta)\right\} - \dfrac{4 C_8}{b} - \dfrac{q b}{2 D} = 0
\end{array}\right\}$$

The specific cases selected for treatment in this paper consider slabs with a free edge boundary, that is

$$M_2(a) = 0 \tag{18}$$

which when applied to Eq. (13) gives

$$\frac{C_6}{a^2}(\mu-1) + 2C_7(1+\mu) + C_8\left[2\ln a(\mu+1) + (3+\mu)\right] + \frac{q a^2}{16 D}(3+\mu) + \alpha(1+\mu)\frac{T}{h} = 0$$

However, other end moments can be handled in a similar manner; Eq. (19) would then equal the end moment rather than zero.

Two conditions still to be specified pertain to the nature of the superimposed loads to be applied to the slab.

Case A—Slab with Free Edge and with Uniformly Distributed Load over Its Entire Surface (Fig. 1)

With the load uniformly distributed over the entire surface the additional two conditions are

$$a)\quad V_1(o) = 0$$

$$b)\quad V_2(a) = 0$$

which gives $C_3 = 0$ and $C_8 = \frac{qa^2}{8D}$ and the solution for the stresses and deflections are obtained in closed form.

For the stress at the center of the slab, Eq. (14) reduces to

$$\sigma_o = \frac{E h}{2(1-\mu^2)}\left[-\frac{C_2 q}{2k\ell^2}(1+\mu) + \alpha(1+\mu)\frac{T}{h}\right] \tag{20}$$

or

$$\sigma_o = \frac{E h}{2(1-\mu)}\left[-\frac{C_2 q}{2k\ell^2} + \alpha\frac{T}{h}\right] \tag{20a}$$

Case B—Slab Subject to Peripheral Load P_o Per Unit Run in Addition to Uniformly Distributed Load over Entire Surface (Fig. 2)

The shear in the center is zero but at r = a, $V_2(a) = -P_o$. Therefore:

$$a)\quad V_1(o) = 0$$

$$b)\quad V_2(a) = -P_o$$

which gives $C_3 = 0$ and $C_8 = -(\frac{Poa}{4D} + \frac{qa^2}{8D})$.

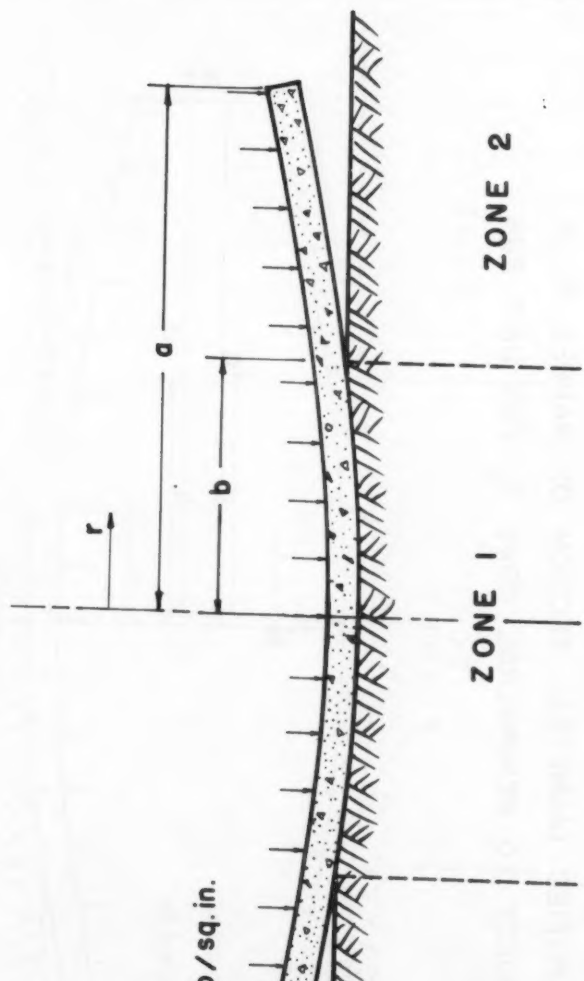
The stress at the center of the slab is given by Eq. (20).

Case C—Slab with Free Edge Subject to Concentrated Load P_1, Acting at the Center of the Slab in Addition to Uniformly Distributed Load Over Entire Surface (Fig. 3)

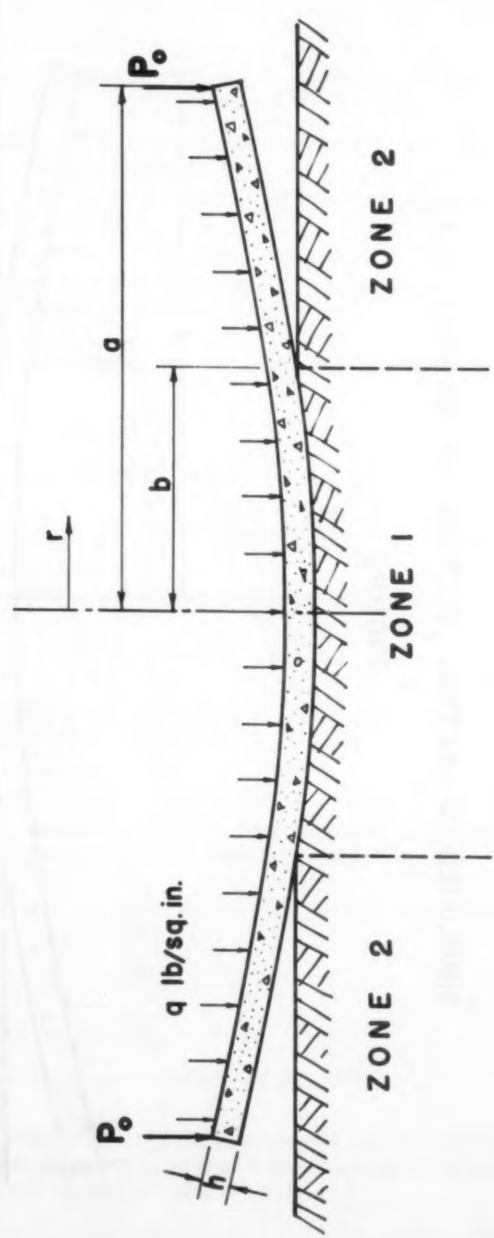
The shear at the edge is zero but at the center the $\lim_{r \to 0}(2\pi r V) = -P_1$

SIMPLIFIED DIAMETRAL SECTION OF WARPED SLAB

FIGURE 1



14



SIMPLIFIED DIAMETRAL SECTION OF WARPED SLAB SUBJECT TO PERIPHERAL LOAD P_o PER UNIT RUN

FIGURE 2

16

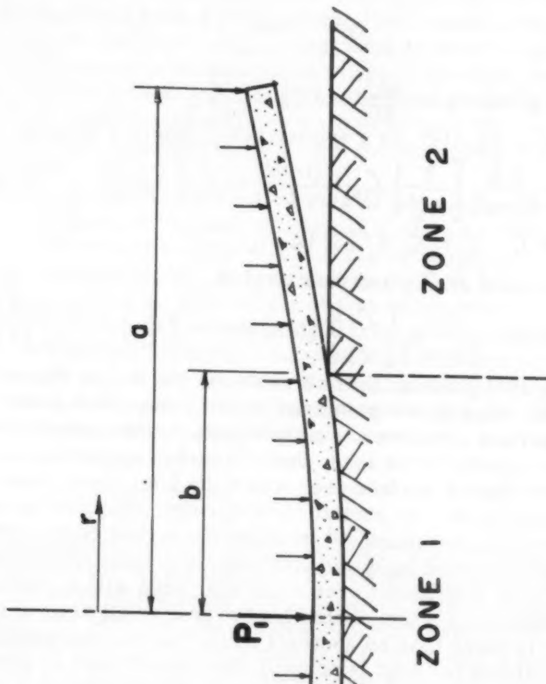


FIGURE 3

SIMPLIFIED DIAMETRAL SECTION OF WARPED SLAB SUBJECT TO CENTRAL CONCENTRATED LOAD P_1

Therefore:

$$a) \quad V_z(a) = 0$$

$$b) \quad \lim_{r \to 0} (2\pi r V) + P_1 = 0$$

which gives $C_8 = -\frac{qa^2}{8D}$ and $C_3 = \frac{P1}{4q\ell^2}$

From Eq. (14), as r approaches ϵ, where ϵ is small, we have[6]

$$\sigma(\epsilon) = \frac{Eh}{2(1-\mu^2)}\left[\frac{q}{k\ell^2}\left\{-C_2\left[1-\frac{(1-\mu)}{2}\right] + C_3\Big[\frac{2}{\pi}\ell_n\frac{\gamma\epsilon}{2\ell} - \frac{(1-\mu)}{\pi}\ell_n\frac{\gamma\epsilon}{2\ell} - \right\} + \alpha(1+\mu)\frac{T}{h}\right]$$

where $\ell_n \gamma = 0.577216$

Substituting for C_3 and rearranging,

$$\sigma(\epsilon) = \frac{Eh}{2(1-\mu^2)}\left[-\frac{C_2 q}{2k\ell^2}(1-\mu) + \alpha(1+\mu)\frac{T}{h}\right] - \frac{3(1+\mu)P_1}{2\pi h^2}\left[\ell_n\frac{\ell}{\epsilon} + 0.1159\right] \quad (21)$$

Eq. (21) gives an infinite value for the stress immediately under the load ($\epsilon = 0$). This is due primarily to the assumption in the derivation of Eq. (1) that normal stresses are proportional to the distance from the neutral axis. In the vicinity of the load, shear stresses apparently increase without limit as the cylindrical surface over which the total shear force P_1 is distributed approaches zero. To obviate the difficulty, recourse is made to the analysis given by Westergaard[(20)] in which the stress on the bottom of the slab under the concentrated load was obtained from thick plate theory by assuming the load to be distributed over a circular area with a radius of 0.325 h.[7] The substitution of a distributed load over a small area in place of a concentrated force is more than an abstract mathematical manipulation. In most practical applications the load is actually distributed over an area and does not, as is often assumed, act at a point. Thus, by substituting $\epsilon = 0.325h$ into Eq. (21) an approximation for the stress at the center of the slab is obtained.

Application

The foregoing theory can be applied to the analysis of a number of problems that arise in practice. As an example, the solution of Case A will be used to analyze the behavior of a concrete pavement slab. Pavement slabs are generally rectangular in shape rather than circular; however, it will be shown later that the shape of the slab may have only a minor influence on the critical stresses or on the deflection pattern.

The particular problem studied involves the determination of the stresses and deflections in a concrete pavement slab subject to temperature and moisture gradients of sufficient magnitude to result in a partially supported slab.[8] Free edge boundary conditions for the slab were believed to be

6. Using assymptotic values for the Z functions.
7. This procedure is based on an analysis given earlier by Nadai.[(10)]
8. As pointed out in the introduction to this paper, temperature and moisture gradients normally encountered in practice are of sufficient magnitude to result in partially supported slabs. This fact has also been demonstrated analytically (cf. discussion of Table I, p. 55).

justifiable as the only loads considered are those due to the weight of the slab; adjacent slabs would be subjected to similar effective gradients, thus resulting in little shear or moment transfer at their edges.

An examination of the equations shows that a prohibitive amount of work would be required to eliminate the constants of integration to obtain an explicit expression for either the stresses or deflections in terms of the slab parameters. Accordingly, the problem was programmed for the Datatron 204 digital computer. As a part of the solution, subroutines were developed for the ber and bei functions and their first derivatives. Fig. 4 is the flow chart for the main program.

The derived equations were solved for a wide range of slab parameters and equivalent temperature gradients (values of Poisson's ratio $\mu = 0.15$ and linear coefficient of thermal expansion $\alpha = 6 \times 10^{-6}$ inches per inch per $^{\circ}$ F. were used throughout the computations). As indicated by the flow diagram (Fig. 4), values of b were assumed for a given set of slab parameters and the corresponding temperature gradients T were obtained. Typical deflection curves for an effective temperature difference between slab surfaces of 30° F are shown in Fig. 5a; Fig. 5b shows the variation in radial (normal) stress across a diametric section. Representative calculated stress relationships are shown in the form of nomographic charts in Figs. 6 and 7.

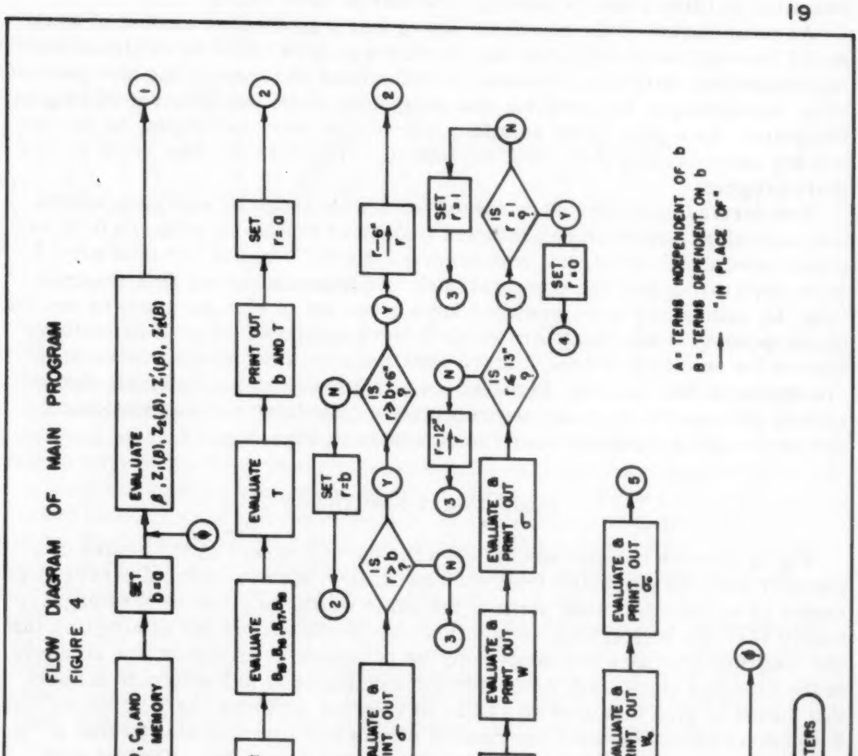
Evaluation of New Theory

Fig. 8 shows a comparison between the normal stress at the center of a circular slab, as computed from the new theory, with the normal stress at the center of an infinitely long strip of the same width, as given by Westergaard's theory.(21) As Westergaard's solution was predicated on the assumption that the subgrade provides full support to the pavement at all times, the stresses in the circular slabs were computed for conditions of full subgrade support. For ratios of slab diameter to radius of relative stiffness ($2a/\ell$) greater than 8, the discrepancy between the central stress in a circular slab to that at the center of an infinitely long strip does not exceed 4 per cent. Typical slab diameters corresponding to $2a/\ell$ ratios of 8 are shown in Table 1 (p. 35). For slab sizes commonly used in practice, it is evident that the critical stresses are not appreciably influenced by the shape of the slab. The equivalent temperature differences (T) shown in Table I are the maximum values that can exist in the slabs if full contact with the subbase is to be maintained. As these temperature differences are much lower than those frequently encountered in practice, the new theory sustains the field observations that concrete slabs on ground are partially supported for a considerable portion of their useful life.

Comparison with Available Measurements

An intensive search of the literature revealed the paucity of useful data on the warping of concrete slabs. Only two sets of data were found in which a sufficient number of slab parameters were measured to permit comparisons between theory and observation.

Fig. 9 shows a comparison of the deflections in a rectangular slab, as measured by Hatt,(5) with those computed from the theory for a circular slab with equivalent parameters. The observed deflections are the result of moisture gradients alone as measurements were made only at times when the



FLOW DIAGRAM OF MAIN PROGRAM
FIGURE 4

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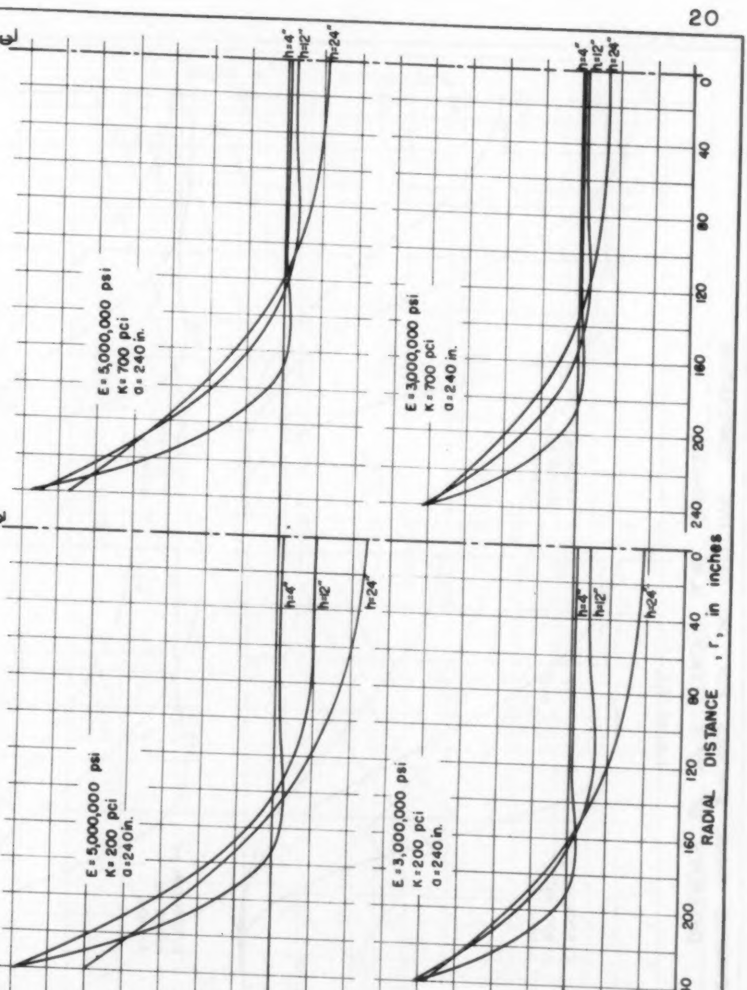

REPRESENTATIVE DEFLECTION CURVES FOR AN EFFECTIVE TEMPERATURE DIFFERENCE OF 30° F. BETWEEN SLAB SURFACES

FIGURE 5a

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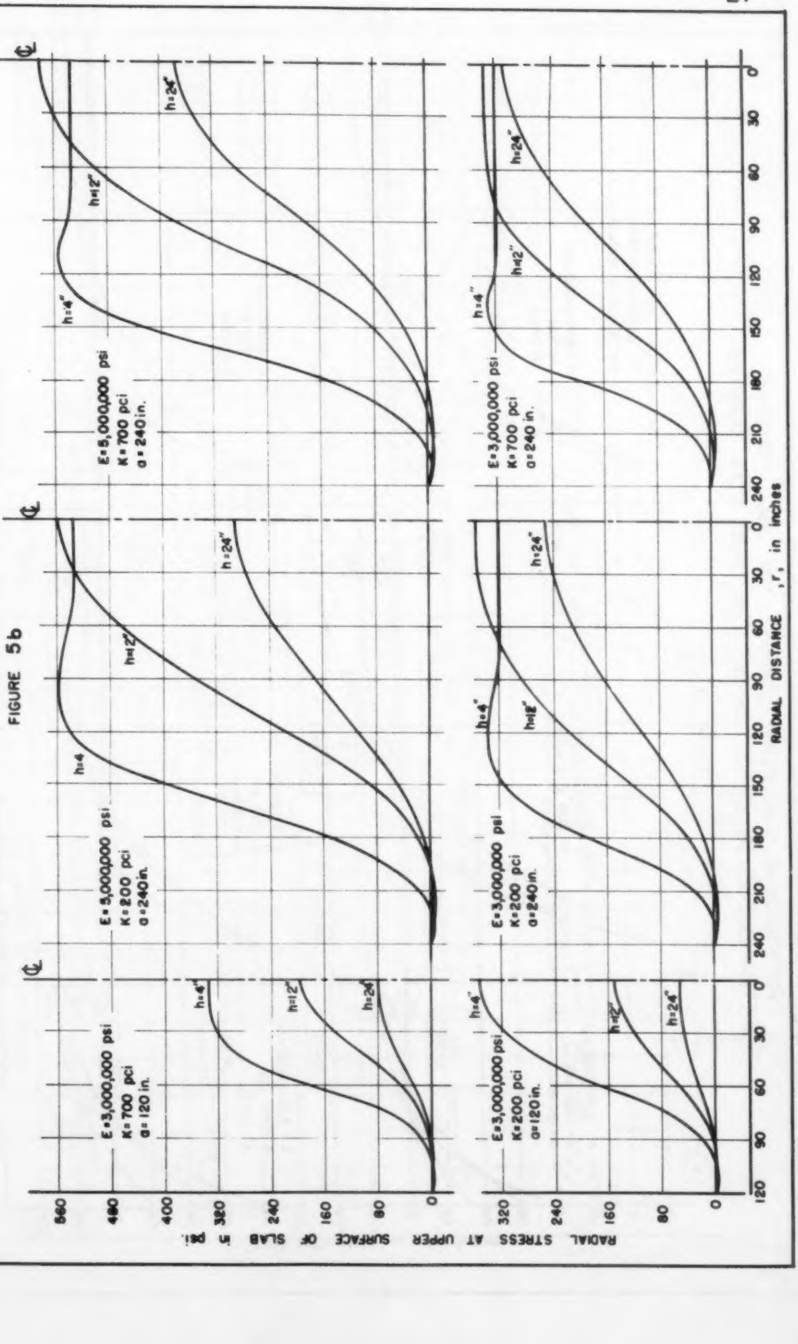

REPRESENTATIVE RADIAL STRESSES FOR AN EFFECTIVE TEMPERATURE DIFFERENCE OF 30° F. BETWEEN SLAB SURFACES

FIGURE 5b

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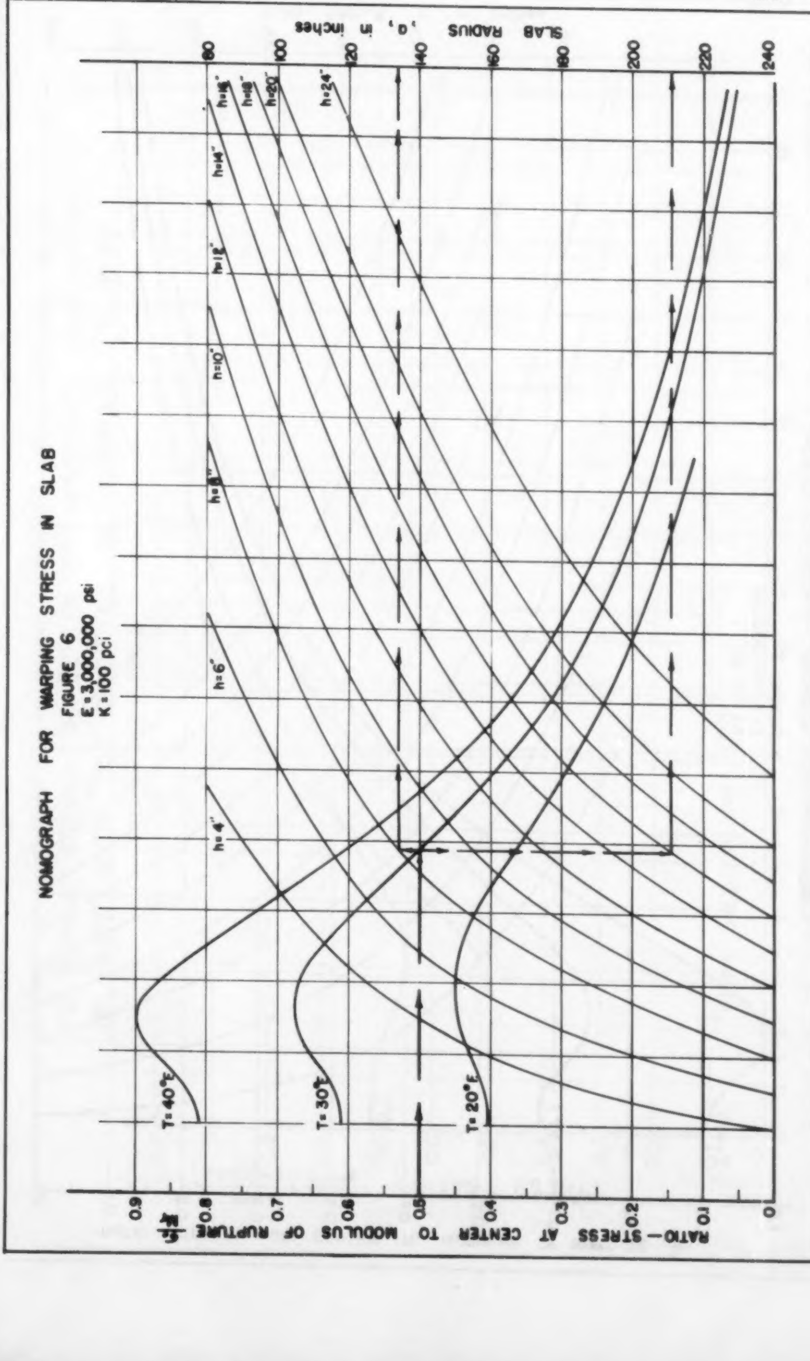
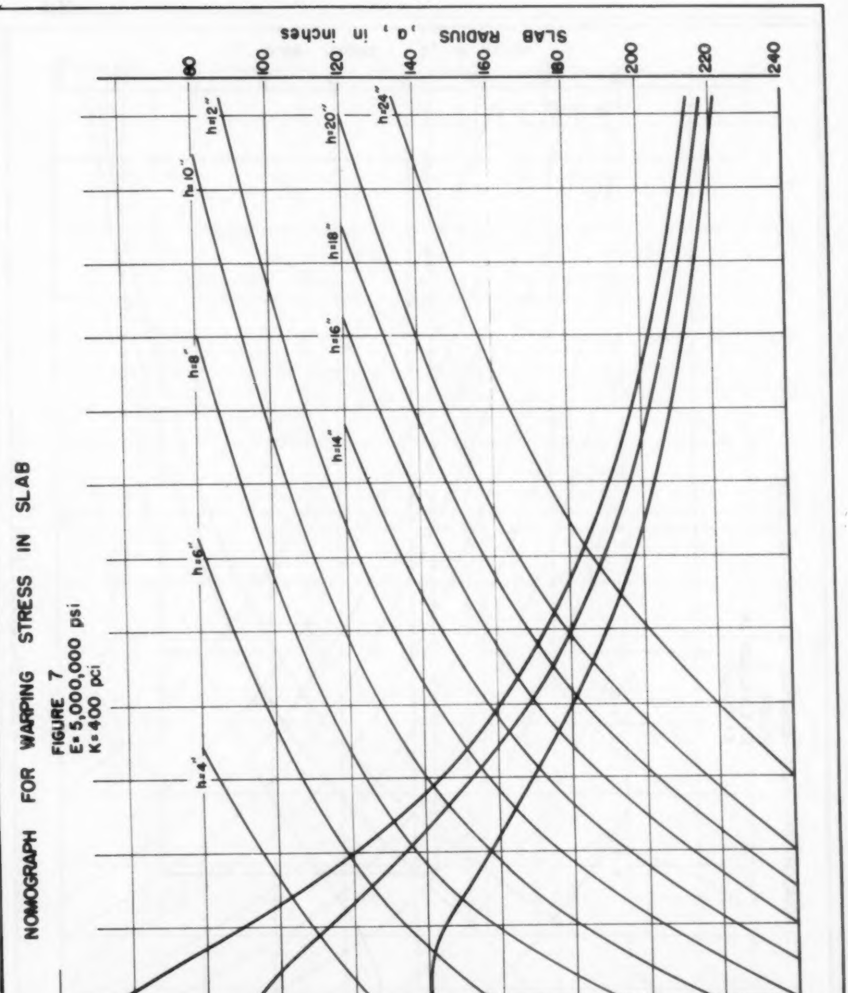


NOMOGRAPH FOR WARPING STRESS IN SLAB
FIGURE 6
E = 3,000,000 psi
K = 100 pci
SLAB RADIUS, a, in inches
80
100
120
140
160
180
200
220
240
RATIO—STRESS AT CENTER TO MODULUS OF RUPTURE $\frac{f_c}{M_r}$
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
T = 40°F
T = 30°F
T = 20°F
h = 4"
h = 6"
h = 8"
h = 10"
h = 12"
h = 14"
h = 16"
h = 18"
h = 20"
h = 24"


NOMOGRAPH FOR WARPING STRESS IN SLAB
FIGURE 7
E= 5,000,000 psi
K= 400 pci
SLAB RADIUS, a, in inches
80
100
120
140
160
180
200
220
240
RATIO—STRESS AT CENTER TO MODULUS OF RUPTURE $\frac{\sigma_c}{M_r}$
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
T=40° F.
T=30° F.
T=20° F.
h=2"
h=4"
h=6"
h=8"
h=10"
h=14"
h=16"
h=18"
h=20"
h=24"

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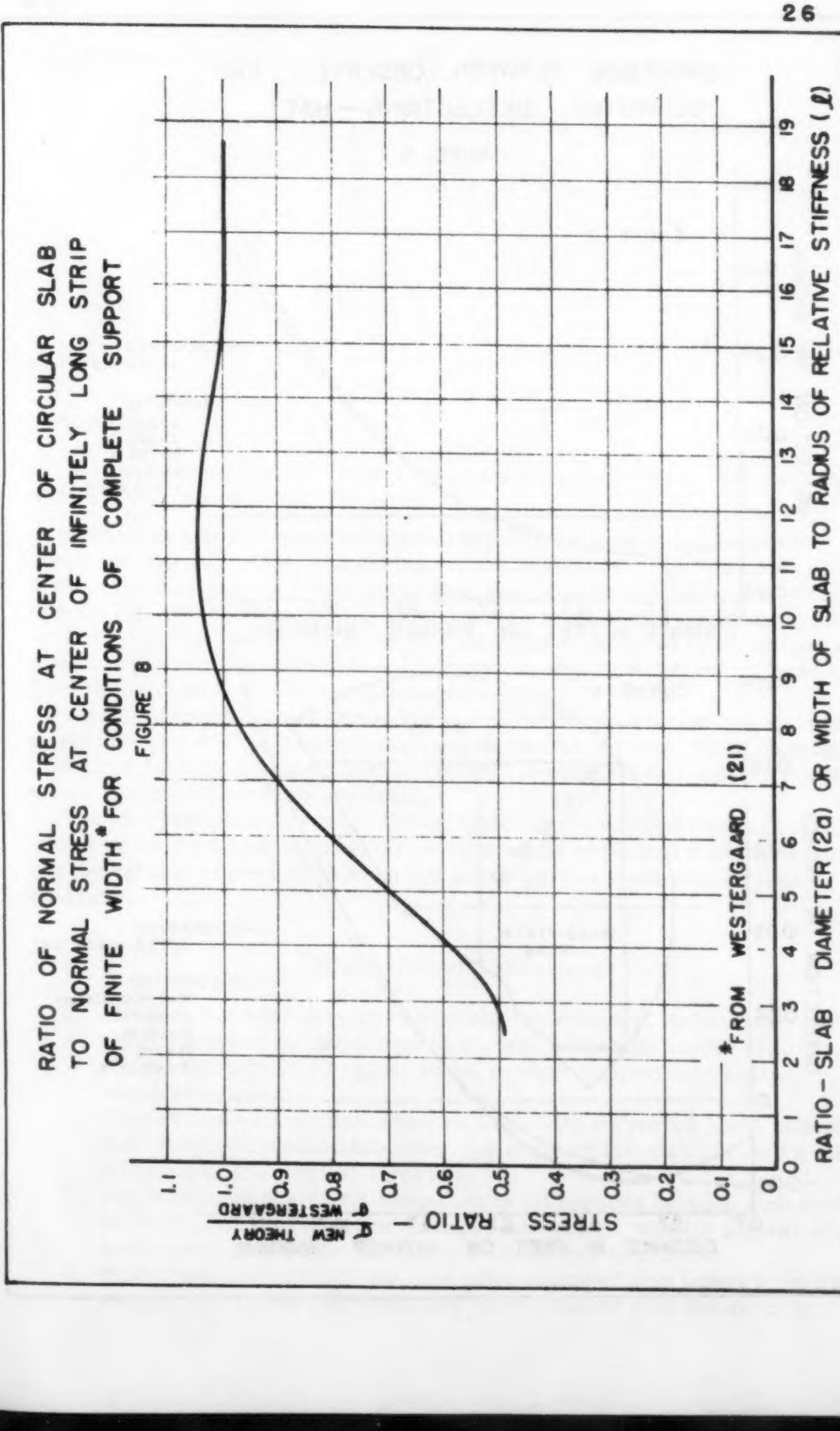

RATIO OF NORMAL STRESS AT CENTER OF CIRCULAR SLAB TO NORMAL STRESS AT CENTER OF INFINITELY LONG STRIP OF FINITE WIDTH* FOR CONDITIONS OF COMPLETE SUPPORT

FIGURE 8

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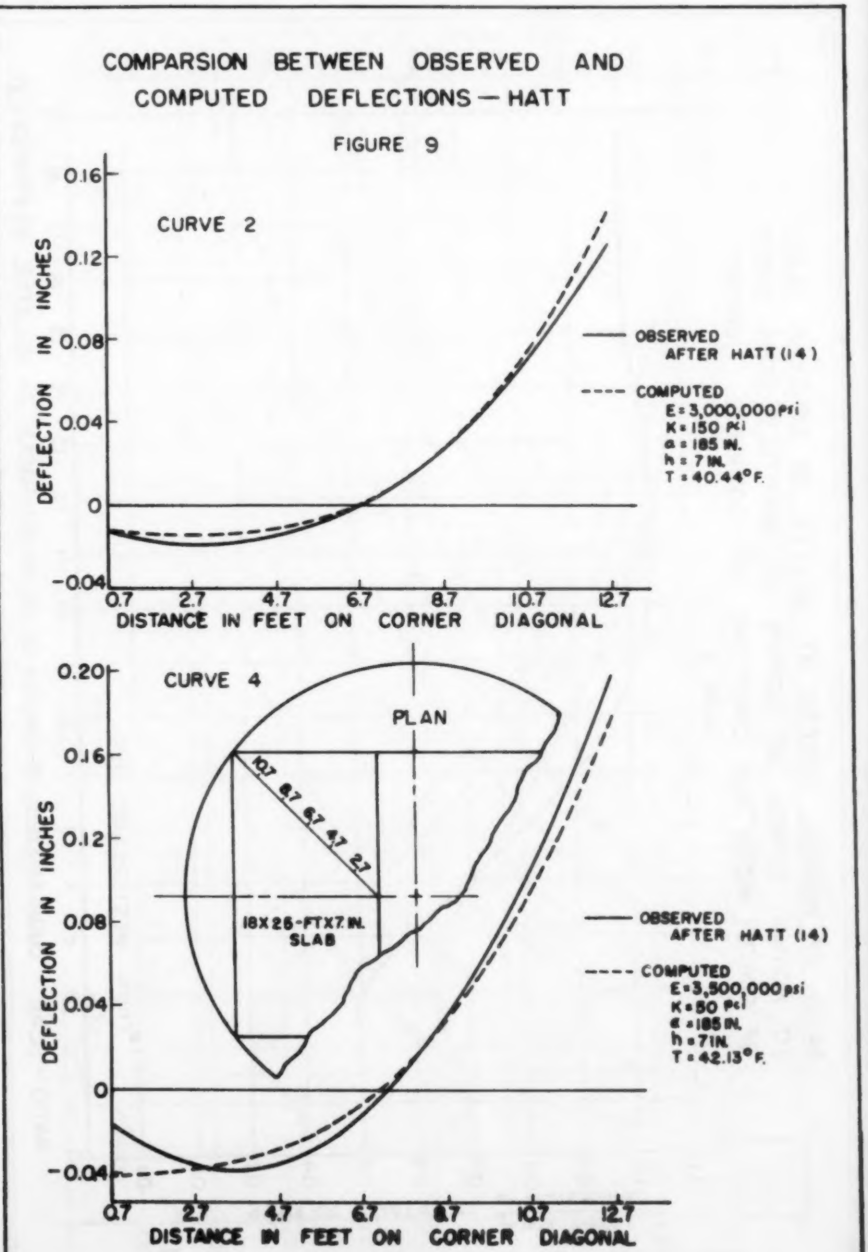

COMPARSION BETWEEN OBSERVED AND COMPUTED DEFLECTIONS — HATT
FIGURE 9
CURVE 2
DEFLECTION IN INCHES
0.16
0.12
0.08
0.04
0
-0.04
0.7
2.7
4.7
6.7
8.7
10.7
12.7
DISTANCE IN FEET ON CORNER DIAGONAL
OBSERVED AFTER HATT (14)
COMPUTED
E = 3,000,000 psi
K = 150 pci
a = 185 IN.
h = 7 IN.
T = 40.44° F.
CURVE 4
PLAN
10.7 8.7 6.7 4.7 2.7
18 X 25 - FT X 7. IN. SLAB
0.20
0.16
0.12
0.08
0.04
0
-0.04
OBSERVED AFTER HATT (14)
COMPUTED
E = 3,500,000 psi
K = 50 pci
a = 185 IN.
h = 7 IN.
T = 42.13° F.

TABLE I

Typical Parameters Corresponding to W/ℓ Ratios of Eight for Full Slab Support*

			K = 200 pci		K = 700 pci	
E	μ	h	W = 2a	T	W = 2a	T
3×10^6 psi	0.15	8"	19.0'	4.36°F	13.9'	2.33°F
3×10^6 psi	0.15	10"	22.4'	4.87°F	16.4'	2.61°F
3×10^6 psi	0.15	12"	25.7'	5.34°F	18.8'	2.85°F

* Note maximum equivalent temperature gradients, T, at which slab can maintain full contact with its support.

temperatures were constant throughout the slab. Curve 2 represents the maximum warping of the slab during the initial drying cycle, whereas curve 4 depicts the maximum position of the slab resulting from first drying the slab completely and then introducing water into the subbase and maintaining its level at the lower surface of the slab. Attention is directed to the computed effective temperature differences (T) shown in Fig. 9, which in this case reflect the influence of moisture gradients only.

Fig. 10 shows a similar comparison with deflection curves obtained at the Portland Cement Association Laboratories, Skokie, Illinois, for a rectangular slab 12' x 24' in plan and 6" thick. The slab in question was subject to both temperature and moisture gradients.

The correspondence between the computed and measured deflections, as shown in the foregoing figures, offers substantial additional evidence regarding the validity of the theory when applied to warped slabs with no edge restraint..

Summary and Conclusions

1. A theory has been developed whereby the stresses, deflections, and degree of support can be computed for symmetrically loaded circular slabs subject to warping caused by ambient temperature and/or moisture gradients.
2. Comparison with available theories for fully supported slabs indicates that, for conventional slab sizes, the shape of the slab has only a minor influence on the critical stresses.
3. The maximum equivalent temperature differences between slab surfaces at which a warped slab can maintain full contact with its support are much lower than those frequently encountered in practice.
4. Numerical solutions for the case of an unloaded slab (apart from its weight) with a free edge boundary were obtained with the aid of a

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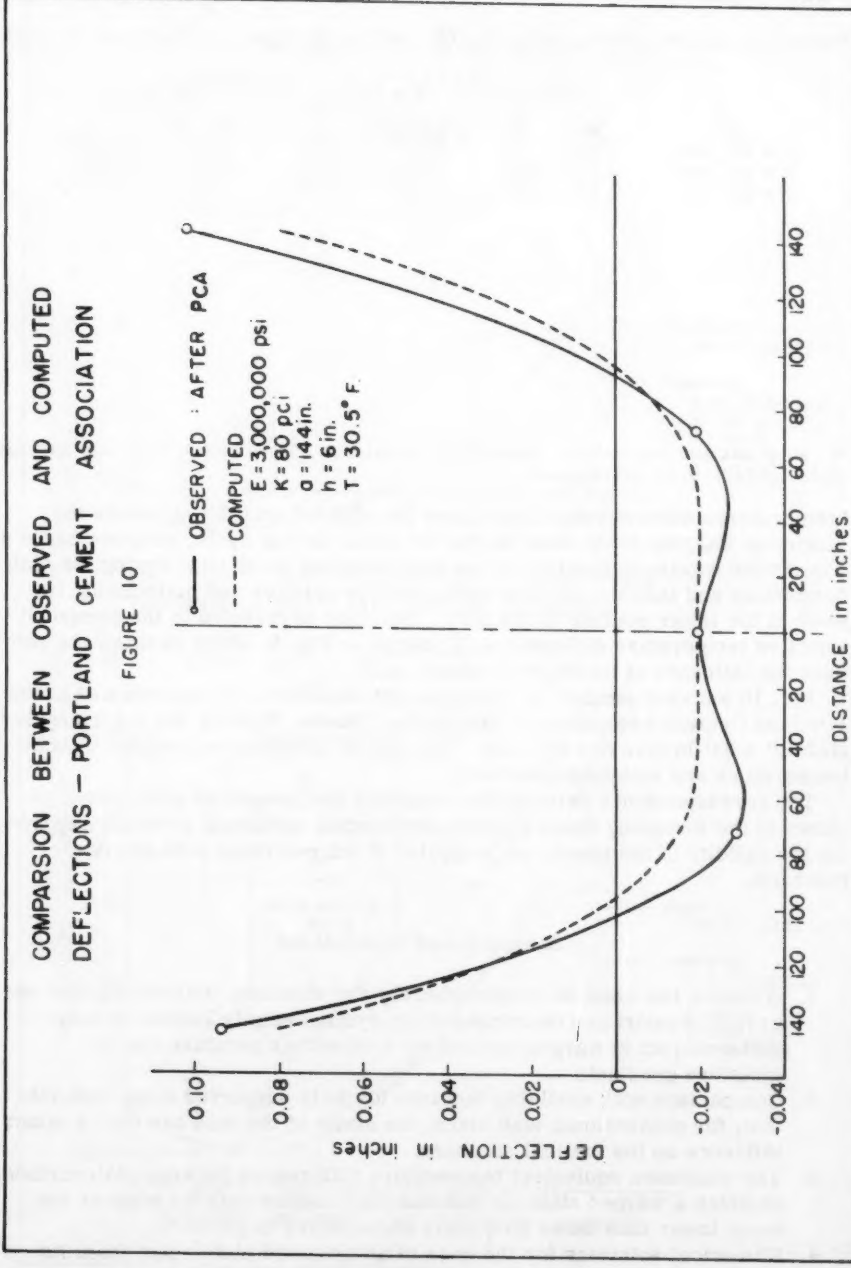

FIGURE 10

COMPARSION BETWEEN OBSERVED AND COMPUTED DEFLECTIONS — PORTLAND CEMENT ASSOCIATION

digital computer. Comparisons with available measurements show, that for this case, the theory can reliably predict the deflections even if warping is sufficiently severe to result in a partially supported slab.

5. The theory shows considerable promise as a useful tool for the analysis of a number of problems concerning the behavior of concrete slabs on ground. Its general validity can be appraised by means of experiments in which temperature and moisture gradients are carefully controlled, and reliable, simultaneous measurements of the deflections, strains, and degree of support are obtained with reference to the initial unwarped condition.

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Proceedings of the American Society of Civil Engineers

CONTENTS

DISCUSSION

(Over)

GROUTING WITH CLAY CEMENT GROUTS[a]

Closure by Stanley J. Johnson

STANLEY J. JOHNSON,[1] M. ASCE.—The laboratory investigation of grouting reported by Mr. Cunny is most informative. It was during a review of the test results from this investigation that the writer first extended Terzaghi's filter ratio concepts to grouting. Mr. Cunny shows that the value of the "Groutability Ratio", which is the ratio of the 15% size of the material being grouted to the 85% size of the grout material, varied between 11 and 24 in the laboratory tests reported. These and other data analyzed by the writer resulted in the tentative criterion that the groutability ratio should generally be in excess of 25 if the grout is to successfully penetrate the formation being grouted.

Mr. Clark's suggestion that the groutability ratio can be as low as 5 is without any known basis to the writer. Extensive filter test data indicate rather conclusively that, in general, a ratio of 5 will guarantee that the soil cannot be grouted. A value of 25 is, however, as Mr. Clark noted, a safe value and lower values (say 15 to 25) apply for some soils and grouting materials. The main value, however, of the groutability ratio is to illustrate the mechanical concept of the size relationship between the voids of the formation being grouted and the size of the particles in the grout. One reason for this is that stratification of the soil in the field is of major importance in determining the success of grouting and is not accounted for when grain size analyses are performed on disturbed samples recovered from borings.

Mr. Clark interpreted too broadly the statement comparing the compressive strength of the grout as determined in the laboratory or in the field and the compressive strength of the grout after injection. His comments are, however, applicable in general, although it would be unusual for a good grout mix to be weaker after injection than the grout as mixed.

The comments by Mr. Clark regarding grouting pressures are not concurred with for the reason that the writer believes that arbitrary pressure limits are more frequently too high than too low. In many instances, large quantities of grout have been wasted because of this. However, as Mr. Clark states, field experiments on pressure are frequently desirable. This statement also is true for almost any phase of grouting work and field experiments should be encouraged.

Mr. Clark's comments regarding Fig. 4 are correct theoretically; however, the intent of the typical envelopes shown is to develop a simple relationship between mixing water and grout proportions, this would not be possible if the small change required to make the values correct for neat cement grout was included.

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a. Proc. Paper 1545, February, 1958, by Stanley J. Johnson.
1. Associate, Moran, Proctor, Mueser & Rutledge, New York, N. Y.

In conclusion, the writer wishes to express his appreciation for the constructive comments and data presented by Messrs. Cunny and Clark.

CEMENT AND CLAY GROUTING OF FOUNDATIONS: PRESSURE GROUTING WITH PACKERS[a]

Closure by Fred H. Lippold

FRED H. LIPPOLD,[1] M. ASCE.—Mr. Clark's suggestion that the controversy over packer versus stage grouting could be settled by using both methods or a combination of the two is a good one. In fact, it is the writer's experience that in actual field practice it is being settled that way. As pointed out, it is not always practical to seat a packer and, even when seated, grout may bypass it through joints in the rock. In such cases, experienced judgment must be used to determine the proper grouting method. If the formation is readily susceptible to lifting, it may be necessary to use a combination of stage and packer grouting, i.e., grout the deeper stages through a packer set in the previously grouted upper stage. In other rock formations this refinement may not be necessary. Foundations that readily accept a thick grout at low pressures can best be grouted initially with a stage method. Packer grouting may be desirable on the final or closure holes. In a relatively tight formation, packer grouting often saves on drilling footage.

Both Mr. Clark and Mr. Schmidt give examples of better packers. The writer did not intend to imply that the packers illustrated were necessarily the best. They were examples of three general types in common use. It is hoped that through usage even better ones will be developed in the future.

a. Proc. Paper 1549, February, 1958, by Fred H. Lippold.

1. Civ. Engr., U. S. Bureau of Reclamation, Denver, Colo.

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PRESSURE GROUTING FINE FISSURES[a]

Closure by Thomas B. Kennedy

THOMAS B. KENNEDY.[1]—The author greatly appreciates the constructive discussion of his paper by Messrs. Clark, Moor, and White.

It should always be kept in mind that it is obviously impossible to duplicate in the laboratory the conditions that prevail in the field. If conditions could be duplicated then the laboratory findings would have direct quantitative as well as qualitative application in the field. Laboratory investigations must be based on standardized, reproducible conditions calculated to develop information and principles that will have general application.

Undoubtedly, there are many cases of effective grouting of seams through the use of thin grouts which lost their excess water, as suggested by Mr. Clark, through a system of interconnected cracks. However, in the absence of interconnected cracks where can the water go?

Mr. Clark's suggestion that neat grout should be passed through the finest practicable sieve is excellent. There is always a certain amount of various oversized material that will be caught on the sieve cloth. Also, the use of a fine sieve provides visual evidence of the thoroughness of the mixing. The use of the finest cement possible and wet sieving should be standard practice when grouting fine cracks.

The writer agrees with Mr. Moor's statement that "total water content of flowing grout is of no particular significance in comparison with water content of grout that has been deposited in ultimate position," but I don't think it is always safe to assume that excess water will be squeezed out under pressure. In many cases pressure sufficient to squeeze out excess water would be damaging to the structure.

Mr. Moor asks whether we were trying to seal the fissures or mate the surfaces bounding the fissures together. We were trying to seal the crack between two concrete slabs and one means for judging the completeness of filling was through observation of the degree of bond between the grout and the underneath surface of the slab above. If there were no contact there could be no bond, neither could the fissure have been completely filled.

Mr. Moor points out need for automatic equipment to indicate and control viscosity of grout in the field. The torque meter is delicate and not too well suited to field use. Devices for detecting changes in viscosity based on electrical demand of mixing equipment might be practical.

The use of lignin salts and other materials can be of value in reducing unit water content without increasing viscosity, however, the physical chemistry

a. Proc. Paper 1731, August, 1958, by Thomas B. Kennedy.

1. Chf., Task Committee on Cement Grouting, Concrete Div., U. S. Army Engr. Waterways Experiment Station, Jackson, Miss.

of their action is not too clear. The American Society for Testing Materials will sponsor a symposium on water-reducing admixtures at the meeting of Committee C-9 on Concrete of that Society in October 1959 in San Francisco. Considerable light should be shed on the mechanism at that symposium.

Both Messrs. Moor and White suggest that grouts with better penetrating qualities will be obtained by using the so-called "colloidal" mixer. This premise appears to be based on the assumption that clumping and floculation occur in neat grout and cannot be overcome by the paddle mixer. I wonder if this is as serious as surmised. Tests made in our own laboratory(1) with a dual-drum "colloidal" mixer indicated that the "colloidal" mixer mixed to about the same homogeneity in about half the time required by the paddle mixer. However, prolonged mixing with the "colloidal" mixer heated the grout and caused rapid increase in viscosity.

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GEOTECHNICAL PROPERTIES OF GLACIAL LAKE CLAYS[a]

Discussion by T. Cameron Kenney

T. CAMERON KENNEY,[1] J. M. ASCE.—In reviewing literature on the subject of shear strength of saturated clays, one is impressed by the amount of excellent work of a practical and fundamental nature which has been done on marine clays, especially the Boston clay, the sensitive Norwegian clays and the overconsolidated English clays. One is also equally impressed by the comparative lack of data on fresh water clays and clay tills. This is a disturbing situation, particularly when consideration is given to the vast areas of the world which are covered with these soils. It is hoped that Professor Wu's paper will be one of the first of a great many articles on these important soils.

This discussion will deal with three aspects of Professor Wu's paper; the apparent relationship between $\frac{c}{p}$ and plasticity index, the shear strength of varved clays and its measurement, and the origin of the unstratified, till-like soils in the Great Lakes region.

1. Relationship Between $\frac{c}{p}$ and Plasticity Index

Much data concerning the undrained shear strength of natural clays has been obtained since the vane came into general use as a piece of field testing equipment. One of the most important things that was found was that the shear strength of a homogeneous deposit of normally consolidated clay increased linearly with depth, or more specifically, the ratio of the undrained shear strength of a clay and the vertical effective stress under which the soil was consolidated in the field was a constant. This ratio has since been denoted by $\frac{c}{p}$. As more data concerning the ratio $\frac{c}{p}$ became available attempts were made to correlate this factor with more fundamental properties of the soil, such as the liquid limit(1) and the plasticity index. It was found that some correlation seemed to exist between this latter parameter and $\frac{c}{p}$ in the case of normally consolidated marine clays.(2) This has proved to be an extremely convenient correlation because it has been confirmed by sufficient field evidence to allow natural shear strengths of marine deposits to be estimated for preliminary design purposes on the basis of the results of simple laboratory tests. However, it must be borne in mind that this correlation between $\frac{c}{p}$ and plasticity index has only been shown to be applicable in the special case of marine clays

a. Proc. Paper 1732, August, 1958, by T. H. Wu.

1. H. G. Acres & Co., Ltd., Consulting Engrs., Niagara Falls, Canada.

and even here recently published data(3) has shown that there possibly is an appreciable range in what was initially considered to be a close relationship.

There are other normally consolidated types of clay which have not been as extensively studied as the European marine clays but whose strength properties are certainly as important; fresh water lacustrine and alluvial clays, glacial clays similar to those in the Great Lakes area in America and in some Scandanavian fjords, and residual clays. The question may be asked —"is the shear strength of these soils related to plasticity index in the same manner as for natural marine clays or is it unrealistic to expect such a general type of relationship to be applicable to all clays?"

It has been shown that the undrained shear strength of clays is dependent upon the structure or particle arrangement, the shear strength parameters, the history of the clay, and the method by which the shear strength is determined. It follows therefore that the ratio $\frac{c}{p}$ is strongly dependent upon the geological history of the soil. On the other hand, the plasticity index is dependent upon the results of two empirical tests on completely remoulded soil; a dynamic strength test (liquid limit) and a type of rheology test (plastic limit), both of which are dependent upon the mineralogy of the constituent particles of the soil. It therefore seems doubtful that any unique and independent relationship exists between these two factors but if any relationships do exist they are probably dependent upon such factors as geological history and test methods.

Empirical correlations, such as $\frac{c}{p}$ -plasticity index, are common in the field of soil mechanics and are extremely useful where great accuracy is not warranted. However using any empirical relationships which has not been shown to have physical justifications is extremely dangerous. (It may be said here that this is not a direct criticism of Professor Wu's paper). For this reason an attempt will be made to express $\frac{c}{p}$ in terms of its dependencies and thus show that it is dependent primarily upon the structure of the soil and the manner in which it is tested.

Let c	denote undrained shear strength.
$(\sigma_1 - \ 3)_f$	denote the deviator stress at failure. Failure is considered to occur when the deviator stress reaches its maximum value.
c', ϕ'	denote the shear strength parameters of the clay.
A_f	denote the value at failure of the pore water pressure parameter relating changes in pore water pressure and deviator stress.(4)

Case I.—If the clay is consolidated under a vertical effective stress p and horizontal radial effective stress $K_c.p$, it can be shown for the case where the direction of the principal stresses remain unchanged that the undrained shear strength can be expressed in the following form:

$$\frac{c}{p} = \frac{1}{2}\frac{(\sigma_1 - \sigma_3)_f}{p}$$

$$= \frac{\frac{c'.\cos\phi'}{p} + \left(K_c + A_f\{1-K_c\}\right)\sin\phi'}{1 + (2A_f - 1)\sin\phi'} \quad (1)$$

For normally consolidated soils c' = 0 and the expression can be simplified to the following form:(5)

$$\frac{c}{p} = \frac{\left(K_c + A_f\{1-K_c\}\right)\sin\phi'}{1 + (2A_f - 1)\sin\phi'} \quad (2)$$

Case II.—In a similar fashion it can be shown that, if a sample is consolidated under the same stress conditions as in Case I but the major and minor principal stresses at failure act in a horizontal direction while the vertical stress remains unchanged, the resulting shear strength can be expressed in the form:

$$\frac{c}{p} = \frac{K_c \,.\, \sin\phi'}{1 + (2A_f - 1)\sin\phi'} \quad (3)$$

In nature, normally consolidated clays generally consolidate under the imposed condition of no lateral yield and for this condition the ratio of the principal stresses is denoted by K_o. Values of K_o have been determined for a limited number of soils ranging in plasticity index from 0 to 45%(6,7) and it was found that for these soils K_o was related to ϕ' in the following manner (Fig. 1):

$$K_o = 1 - \sin\phi \quad (4)$$

Substituting this expression into Eqs. (2) and (3), the following relationships are obtained:

Case I

$$\frac{c}{p} = \frac{(1 - \sin\phi' + A_f \,.\, \sin\phi')\sin\phi'}{1 + (2A_f - 1)\sin\phi'} \quad (5)$$

Case II

$$\frac{c}{p} = \frac{(1 - \sin\phi')\sin\phi'}{1 + (2A_f - 1)\sin\phi'} \quad (6)$$

Eq. (5) represents the strength which would be measured in the laboratory if a sample was consolidated under an axial stress of p for the condition of no lateral yield and was failed in compression by increasing the axial load. It also represents the strength which would be measured in a laboratory compression test on a sample which had been consolidated in-situ under a vertical effective stress p and had been sampled without causing any disturbance to the soil structure. Eq. (6) represents the strength which would be mobilized in a vane test if the shear strength mobilized along the end surfaces was equal to the shear strength mobilized along the vertical cylindrical surface; this

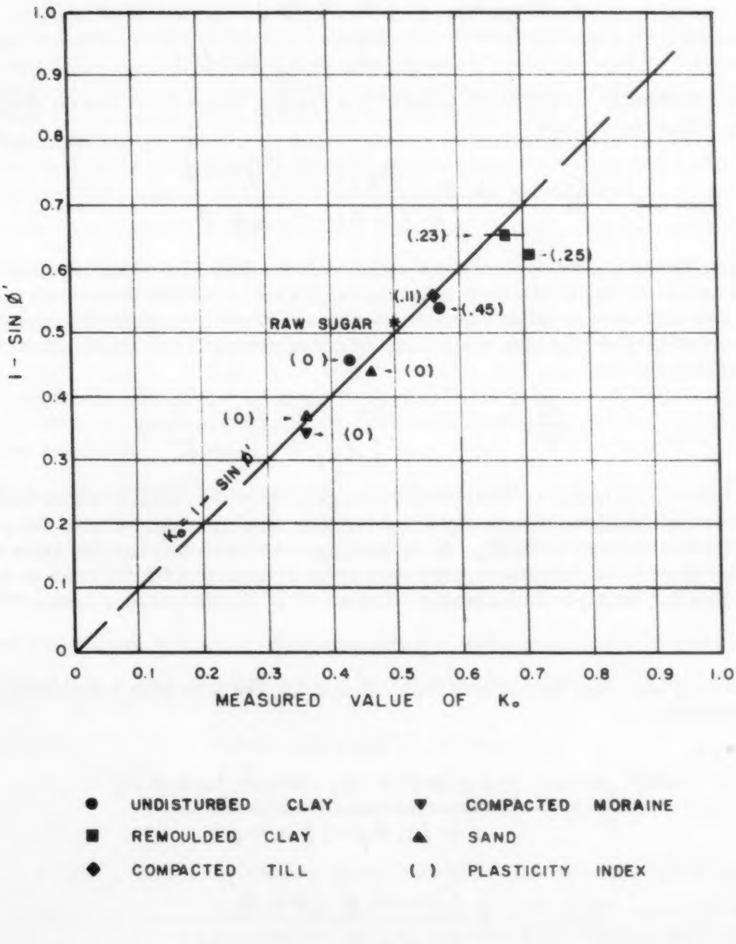

FIGURE 1 - RELATIONSHIP BETWEEN K_o AND ϕ'

assumption introduces a small error since the shear strength mobilized along the end surfaces is higher.

It is thus seen that the value of $\frac{c}{p}$ measured by means of compression tests and vane tests is dependent upon two parameters, the pore water pressure parameter A_f and the shear strength parameter ϕ'. The value of A_f for any soil is dependent upon the rigidity and resiliency of the clay structure and upon the type of test used to cause sample failure. In Table I values of A_f are listed for various natural and remoulded soils. These values were all measured in triaxial compression tests; the writer knows of no reference where values of A_f have been published for tests other than axial compression tests. Unfortunately, the unstable structure of sensitive clays partially collapses during reconsolidation in the laboratory and thus it is very likely that the values of A_f for the natural sensitive soils noted in Table I are smaller than the values of A_f for the clays in their natural state.[8] It may be noted that the value of A_f varies at least from 0.26 to 2.4, that the more sensitive soils have the higher values of A_f, and that A_f is independent of plasticity index.

In order that the possible ranges of $\frac{c}{p}$ for various values plasticity index could be shown graphically, a correlation between sin ϕ' and plasticity is used for convenience* (Fig. 2). There is an obvious scatter of points and it is seen that the less active soils exhibit higher values of ϕ'. The test data are results of drained tests and undrained tests with pore water pressure measurements on normally consolidated natural and remoulded marine, fresh water and residual clays. Possible values of $\frac{c}{p}$ for conditions of failure implicit in Eqs. (5) and (6) have been calculated for several values of A_f by using the relationship between sin ϕ' and plasticity index indicated by the dashed line in Fig. 2. These values are plotted in Fig. 3. It is seen that theoretically $\frac{c}{p}$ is essentially independent of ϕ' and therefore independent of plasticity index and is primarily dependent upon the type of test and the magnitude of A_f. Since the magnitude of A_f for a certain type of test reflects the rigidity and the resiliency of the soil structure, and since these properties are dependent upon the method of deposition of the soil particles and the nature of any subsequent physio-chemical changes in these particles, it follows that A_f and therefore $\frac{c}{p}$ is probably chiefly dependent upon the geological history of the clay. The results of field vane tests have also been plotted in Fig. 3. The broad scatter of these points, even for clays of the same geological origin, seems to confirm the possibility that $\frac{c}{p}$ is not necessarily dependent upon plasticity index. (An interesting sidelight of Fig. 3 is the fact that shear strengths measured by means of the vane test will not generally be equal to the shear strengths measured by means of compression tests on sampled soil;[9] in order to make these two results correspond more closely the vane equipment is generally

*This correlation is independent of soil structure. It has been found (unpublished data) that the angle of shearing resistance, ϕ', for normally consolidated soils is essentially the same for both undisturbed and fully remoulded samples.

Table I Values of A_f for Some Normally Consolidated Soils

Reference	Clay	Type	w_L	Ip	St	Kc	A_f	ϕ'
Natural Soils								
Bjerrum & Rosenqvist (8)...	Toyen...........	Marine.....	47	25	8	0.52	1.50	28.5
			47	25	8	0.60	1.48	
	Drammen..........	Marine.....	36	16	4	1.00	1.2	35.0
			36	16	4	0.68	2.4	
Taylor (15)................	Saco River.......	Marine.....	46	17	10	1.00	0.95	32.5
Taylor (19)................	Boston...........	Marine.....	-	-	-	1.00	0.85	34.6
Unpublished................	Bersimis.........	Estuarine..	39	18	6	1.00	0.63	38.7
Bishop & Henkel (16) ..... Skempton & Bishop (17)	Chew Stoke.......	Alluvial...	28	10	-	0.43	0.59	33
Unpublished................	Kapuskasing......	Lacustrine.	39	23	4	1.00	0.46	30.0
Taylor (15)................	Decomposed Talus.	Residual...	50	18	1	1.00	0.29	34.6
Unpublished................	St.Catharines....	Till (?)...	49	28	3	1.00	0.26	25.6
Remoulded Soils								
Henkel (18)................	London...........	Marine.....	78	52	1	1.00	0.97	19
	Weald............	Marine.....	43	25	1	1.00	0.95	23
Unpublished................	Beauharnois......	Till (?)...	44	24	1	1.00	0.73	30.5
Taylor (15)................	Boston...........	Marine.....	48	24	1	1.00	0.69	30.7
Unpublished................	Beauharnois......	Estuarine..	70	42	1	1.00	0.65	32.8
Unpublished................	Bersimis.........	Estuarine..	33	13	1	1.00	0.38	39.0

w_L - liquid limit, Ip - plasticity index, St - sensitivity

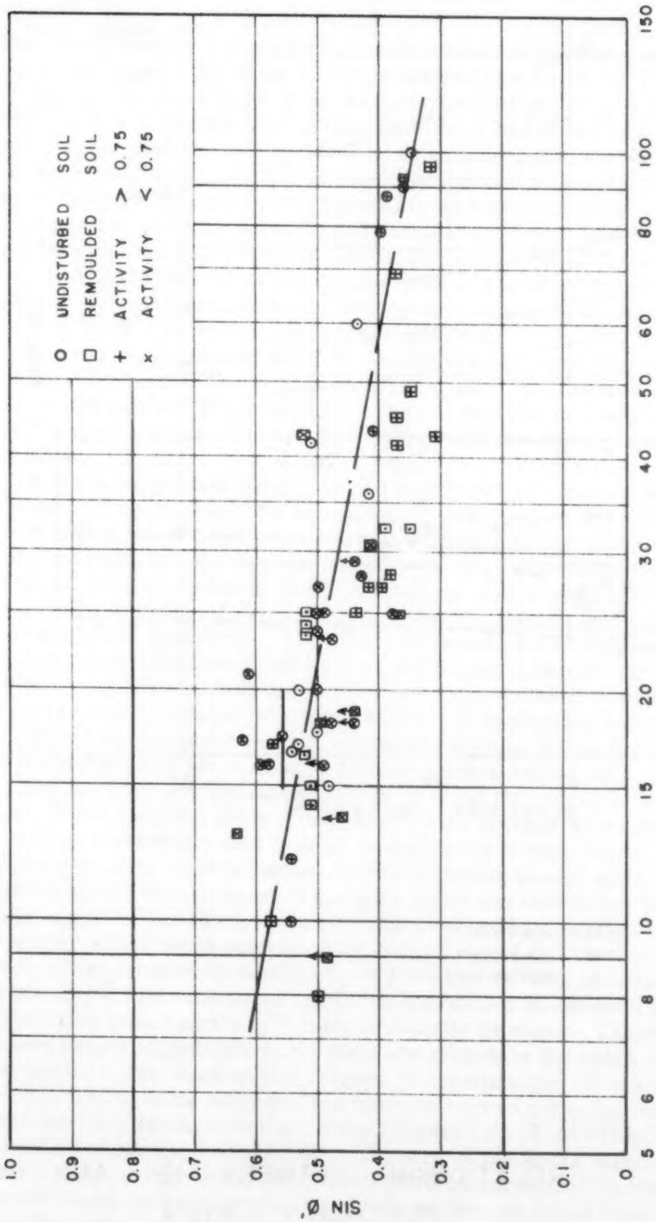

FIGURE 2 - RELATIONSHIP BETWEEN SIN Ø' AND PLASTICITY INDEX

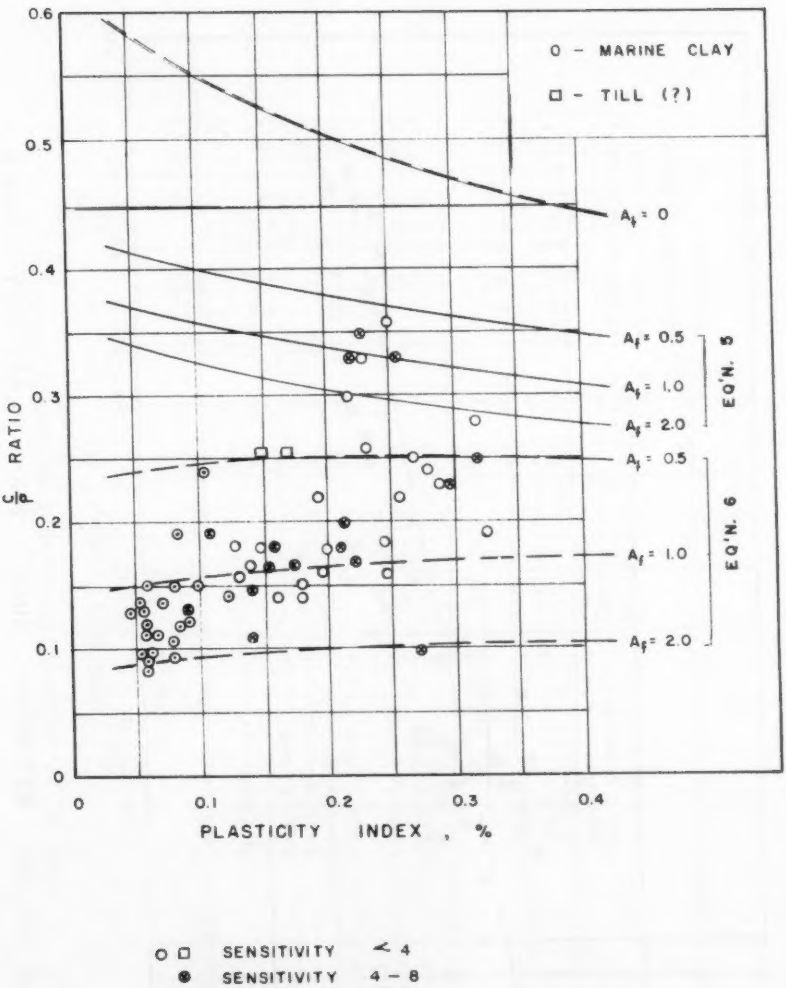

FIGURE 3 - RELATIONSHIP BETWEEN $\frac{c}{p}$ AND PLASTICITY INDEX

calibrated and the rate of rotation of the vane is varied to suit).

2. Varved Clays

The characteristic which distinguishes varved clays from other clay deposits is the regular variation in materials within a small vertical distance which has been caused by cyclical deposition. This structural feature of varved clays is of fundamental importance and a knowledge of the in-situ variations in properties throughout these soils is a necessity for clear understanding of their behaviour. The geotechnical properties of the different soil layers in these deposits have received comparatively little attention, especially such characteristics as shear strength and consolidation, and this fact can probably be attributed to the difficulties in testing techniques which these soils present. In Fig. 4 is shown the variations in plasticity and activity for different laminae in several varved clays. The fact that we are dealing with layers of materials which vary widely in properties is quite apparent.

In his treatment of these soils Professor Wu has unfortunately chosen to deal with them as if they were homogeneous soils. He has quoted Atterberg limits, water content, grain size distribution, particle orientation, chemical properties and associated properties without making reference to the particular lamina of the varve to which the properties correspond or without stating whether these properties were obtained from tests on mixed samples of several laminae. It is realized that in some cases the varves are too thin to separate out and in this case it seems only practicable to use mixed samples. The results in Professor Wu's paper would be of more value if he would specify the type of lamina (coarse or fine) to which the test results correspond.

The shear strength of varved clays is a subject which warrants a great deal of study and research. The subject is complicated by the fact that the soil consists of layers with different shear strength, stress-strain, and consolidation properties. Thus far it has been found difficult to establish the correct laboratory testing procedure which will reproduce conditions and give strengths which are in agreement with those existing in the field. In the field, failure usually takes place along a smooth surface rather than a broken or irregular surface, and the soil everywhere along this surface fails in shear (except, of course, that portion near the ground surface where a tension crack may form). To produce this type of failure in the laboratory or in a field test, the failure surface must be made planer. In homogeneous soils a planar type of failure should always occur if the pore water pressures are positive or only slightly negative, but in varved clays where the soil properties may change appreciably within short distances the sample might fail along an irregular surface in the process of seeking out the surface of least resistance. It has been shown(10) that this can occur in the unconfined compression test where samples have failed partly in tension and partly in shear. This type of failure is kinematically impossible in the field and therefore the value of any test in which failure takes place in this manner is questionable. It has also been shown(11) that in some compression tests on varved clays the material of the weaker laminae has squeezed out from between two of the stronger laminae. The results of such tests would be a measure of the shear strength of the weaker lamina, but if analyzed as a standard compression test the resulting value for shear strength would represent neither the actual shear strength of the weaker layer nor that of the varved sample.

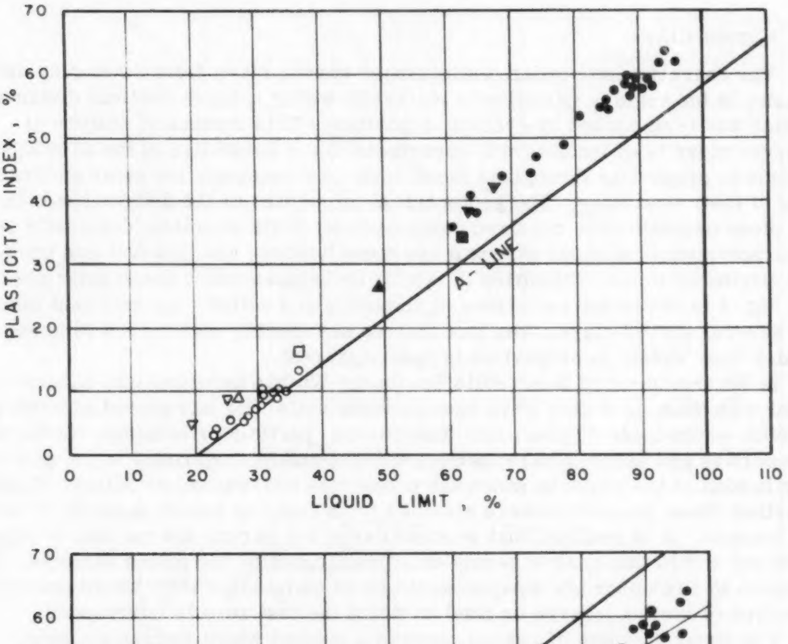

FIGURE 4 - VARIATION IN PLASTICITY AND ACTIVITY OF LAMINAE IN SOME VARVED CLAYS

If the shearing surface was planar the shear strength of the soil would still be dependent upon the orientation of the shear surface relative to the varves. If failure takes place wholly within one lamina, the shear strength mobilized along the failure surface would be expected to equal the shear strength of the soil making up that lamina. If, however, failure takes place across the varves, it can be shown that, for differing values of the shear strength, strain at failure, and thickness of the different laminae, the mobilized shear strength may vary from a value less than the shear strength of the weaker layer to a value greater than the shear strength of the weaker layer, but always less than the shear strength of the stronger layer.

It follows, therefore, from the above discussion that the simple compression test is probably an unsatisfactory method of measuring the undrained shear strength of varved clays. The type of laboratory test which appears to satisfy the requirements best is the shear box test in which the sample fails in direct shear and in which the orientation of the failure plane can be controlled. Some difficulties might be encountered in controlling the drainage conditions satisfactorily in this test. The field vane test is another test in which the failure plane is controlled but this test would measure only the composite strength of several laminae.

In his treatment of the subject of undrained shear strength of varved clays Professor Wu has used the unconfined compression test to determine shear strength. It would be appreciated if Professor Wu would comment upon the type of failure which was observed in his test work on varved clays in view of the effect that the type of failure might have upon the test results if the tests were analyzed in the standard manner.

It may be noted that the shear strength of the varved clay at Sault Ste. Marie was measured in the field by means of the vane and in the laboratory by means of the unconfined compression test.(12) The plasticity index of the laminae of the curves varied at least from 16 to 34 per cent. The value of $\frac{c}{p}$ found by means of the vane was equal to 0.34 while the value of $\frac{c}{p}$ found by means of the unconfine compression test was equal to 0.24. This great difference may be partially due to testing partially disturbed samples in laboratory but it also may be due to the type of failure which occurred in the unconfined compression tests. For somewhat similar varved clay in the area of Sault Ste. Marie, Professor Wu obtained a value of $\frac{c}{p}$ equal to approximately 0.15 by means of the unconfined compression test.

3. Till

The second interesting soil type with which Professor Wu dealt is the soil which he described as "unstratified silty clay with some sand and gravel, the texture of which closely resembles that of till". This soil type is extremely interesting from a geological point of view for several reasons:

(a) Deposits of this type extend over much of the Great Lakes area from Hamilton, Ontario to Chicago, Illinois.
(b) Some of these deposits are extremely homogeneous over depths of 100 feet.
(c) These soils are generally normally consolidated.
(d) These soils have sensitivity values equal to approximately 3.

(e) The fabric or structure of the clay portion of these soils is similar to the structure of a soil sedimented in a deflocculated state and is also similar to the structure of a remoulded soil.(13)

If it is supposed that this material is till, the following questions arise—why is the soil so homogeneous in texture, and how was it deposited in a normally consolidated state? If it is supposed that this soil is a lacustrine deposit, the following questions arise—why is the material so homogeneous with respect to particle size gradation and why does it contain a wide range of particle sizes from clay to gravel? It would be appreciated if Professor Wu would comment upon the geological history of these soils. It may be noted that a similar material was found in a Norwegian fjord to a depth of at least 20 feet and possibly as great as 80 feet and its formation is thought to have been due to ice rafting.(14)

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COMPUTATION OF THE STABILITY OF SLOPES[a]

Discussion by Marcel Bitoun

MARCEL BITOUN,[1] A. M. ASCE.—The author's method of determining directly the most critical failure circle is very interesting. However, it is doubtful that the application of this theory can be extended beyond the restricted field of simple cases. The conventional slip circle analysis with trials will still be resorted to for the solution of problems involving zoned embankments, for instance.

The developments presented by the author suggest a few interesting remarks: in the case of cohesionless materials (C = 0), the expression of the factor of safety becomes:

$$F_g \doteq f_2 \frac{\tan \phi}{\tan i}$$

This shows that the location of the critical circle and the corresponding value of the minimum safety factor are independent of the height of the embankment. The function f_2 is minimum and equal to 1 for y = 0. The safety factor then becomes equal to:

$$F_g = \frac{\tan \phi}{\tan i}$$

The arc offering the least resistance to failure is then infinitely close to the surface of the slope, which translates the fact that the stresses are maximum at the surface.

The safety factor of stability of any slope made of dry granular material could thus be found from the following graph.

In the case of a sudden draw-down, for an embankment made of cohesionless material which would not be perfectly free-draining (case of ϕ equal to 20 to 30 degrees), if w" is the submerged unit weight, and w' the saturated unit weight of the material, the factor of safety will be equal to:

$$F'_g = \frac{w''}{w'} \frac{\tan \phi}{\tan i} f_2$$

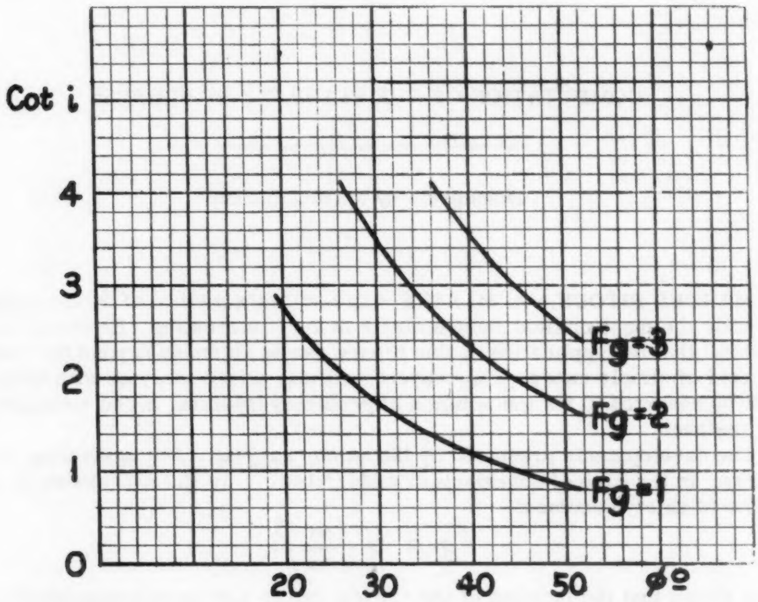
if one introduces the submerged unit weight in the determination of the sum of the normal forces. Then:

$$F'_g = \frac{w''}{w'} F_g$$

The safety factor is therefore reduced in the same ratio as the unit weights.

a. Proc. Paper 1824, October, 1958, by Otto H. Meyer.

1. Chf. Design Branch, Div. of Flood Control, Pennsylvania Dept. of Forests and Waters, Harrisburg, Pa.



These results, simple as they are, are evidently based on the assumption that failure occurs along an arc of circle, which is not necessarily true. Other assumptions have been advanced: wedge, logarithmic spiral, ... However, it seems that the previous results could be used for preliminary studies of slopes made of granular material.

GLOSSARY OF TERMS AND DEFINITIONS IN SOIL MECHANICS[a]

Discussion by R. F. Legget

R. F. LEGGET,[1] M. ASCE.—The two committees responsible for this glossary are to be heartily congratulated on achieving such excellent progress in the difficult assignment which has been theirs. All engaged in soil mechanics work will be pleased at the prospect of having an authoritative glossary of terms and definitions for use in writing in this field. The final completion of this important project, so happily carried out jointly between ASCE and ASTM, will therefore be cordially and widely welcomed.

All who have had to prepare such glossaries know the difficulties involved. Accordingly, it would be invidious to make any comments upon this excellent final draft were this not the express wish of the committees themselves. In view of this specific invitation for discussion the writer therefore submits the following suggestions, in the hope that they may be found to be constructive and that they may possibly be of some use in finalizing this important project. For convenience comments are grouped into those that are of relatively minor importance and others which appear to warrant some more serious consideration. The numbers used refer to the paragraphs in the paper as published.

Minor Comments

56. The inclusion of 'e' (void ratio) as a symbol directs attention to the misprint in the formula ('3' for 'e').

63. The word 'of' in the first line should be 'on'. The final sentence in this definition strikes an odd note since no other uses and applications are included in the glossary; if one use is given, why not many others?

92. Is not the expression "unit of pressure" incorrect, since pressure is normally expressed as a load per unit area?

97. The last sentence in the definition does not seem to add anything useful and might be omitted.

103. The inclusion of this term is questioned since it has not come into general use, despite the advocacy of Dr. Kirk Bryan; if it must be included then a much more precise definition than that given is desirable.

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a. Proc. Paper 1826, October, 1958, by the Committee on Glossary of Terms and Definition in Soil Mechanics of the Soil Mechanics and Foundations Division.

1. Director, Division of Building Research, National Research Council, Ottawa, Ont., Canada.

120. The wording of this paragraph is puzzling since the word 'condition' does not seem to relate to anything which precedes it. Some re-writing seems to be necessary.

122. The same comment applies as to paragraph 120.

Major Comments

11. This definition seems to be incomplete without the inclusion of the words 'safety of a' between 'the' and 'structure' in the last line. Without these additional words the definition has little real meaning.

12. Correspondingly the word 'endangered' in the last line of this definition is one which must be defined if the definition is to be useful; since the safety of structures is involved in both these definitions they appear to the writer to be of singular importance.

67. The definition suggested for cohesion seems to be quite unsatisfactory by being related only to a term in the equation and not to a physical property of soil.

73. In this and some later definitions the only use of a personal name is included by reference to the work of Mr. R. R. Proctor. The writer is sure that Mr. Proctor would be the first to object to this distinction. May it therefore be suggested that the word "Proctor" be omitted from all definitions relating to compaction, this suggestion being made with the more emphasis since the compaction study so commonly associated with the name of Mr. Proctor had been carried out almost simultaneously by Mr. A. A. Kelso in Australia.*

80. With great respect the writer questions the accuracy of the definition suggested for 'Consistency'.

107. This definition needs drastic rewording since degree days are certainly not "the difference between the average temperature each day and 32^{0} F". They can more properly be defined as the summation of the products of days and the difference between the average temperature each day and a stated temperature. The most commonly used degree days are those based on a temperature of 65^{0} F, the use of 32^{0} F as the base temperature having a much more restricted use.

152. The definition of 'footing' does not seem to be very satisfactory since there are many 'foundations' which transmit loads to the soil without the aid of footings.

153. One wonders why the word 'earth' is used here in distinction to the word 'soil', which is used in the preceding line and which is defined in the glossary.

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*Kelso, A. A. "The Construction of Sylvan Dam, Melbourne Water-Supply." Proc. Inst. Civil Engrs., Vol. 239, pp. 407-409, May 1936 (Compaction information on pages 407-409; Paper delivered in London on 8 January 1935.

170. The writer regrets to an unusual degree inclusion of the word 'hardpan' in any glossary to be sponsored by the ASCE. This is a most dangerous word, the use of which should be abandoned in all civil engineering circles. The only satisfactory definition of 'hardpan' which the writer has ever met is that it is material which proved harder to excavate than the contractor anticipated. As such it has no place in a technical glossary.

230. The definition of 'muskeg' does not seem to be satisfactory in the light of recent research into muskeg and its properties. The recent paper by Mr. MacFarlane (published in the ASCE Soil Mechanics Journal) contains references which might assist the committees to devise a better definition.

244. A similar comment applies here to the definition of 'peat'.

245.
246.
285. In all these definitions the word 'Proctor' appears and it is suggested that it should be omitted.
286.
287.

252. The definition of 'permafrost' needs a good deal of qualification since the word is generally used to denote the condition in which the ground is at a temperature below 32° F. The word is also used in a more popular sense, to describe perennially frozen soil but its wider use certainly calls for reference.

258. The definition of 'piezometer' seems to be the least satisfactory of those in the glossary; is it really necessary to include the name of such a simple measuring device in such a glossary?

316. The writer found difficulty in appreciating the definition suggested for 'shear failure' particularly since it includes the suggestion of "destroying a structure". With respect, it is suggested that this definition be re-written with the omission of any reference to structures since the shear failure surely is a phenomenon in a soil mass, irrespective of what may be above the surface of the soil.

330. The writer can well imagine that the committee had much discussion about this definition of 'soil', and therefore refrains from further critical comment, even though he does not find the definition too satisfactory; if it must stand as written then he would suggest that the second word 'or' be changed to 'and'.

332. The use of the expression 'soil profile' will not be meaningful to those who use the glossary unless they are familiar with pedological studies; it is suggested that this expression might be replaced.

393. This definition might well be described as an "open ended" definition, but admittedly it is difficult to be precise about the meaning of 'undisturbed samples'. If the committee decide to leave this definition as it stands then the writer would suggest that the third word from the end should be changed from 'to' to 'of'.

The writer regrets the undue length of these comments now that they are recorded on paper, but trusts that the committees will take them as an indication of the great interest which the glossary has provided for him.

SETTLEMENT OF OIL STORAGE TANKS[a]

Discussions by John A. Focht, Jr. and R. G. Brickell and A. W. Smith

JOHN A. FOCHT, Jr.,[1] A. M. ASCE.—It is indeed gratifying that publication has been made of observed tank settlements which are of considerable magnitude but which have not adversely affected operation of the tanks. It is further gratifying that the data was assembled by one of the major oil companies, which gives authority to the observations. Too frequently demands have been made of soils engineers that tank foundations must have negligible settlement. Again too frequently actual measurements are not made to determine the performance of tanks or other structures; reliance is often placed in the general passing observation that no distress is evident in the tank shell or piping connections.

In the discussions of foundation ring walls, brief mention is made that "in some instances, the foundation ring had failed from lateral pressure." Were any analyses made using the dimensions of the failed rings to estimate the actual lateral pressure? The longitudinal reinforcing steel in the Type "B" foundation ring is stated to be sufficient to carry the hoop stresses created by lateral soil pressure. It would be interesting to know what procedure the author employs to estimate the lateral earth pressure on the ring wall and his basis for selection of that procedure.

The author and his employer are to be commended for making the detailed settlement observations and for releasing them to the profession.

R. G. BRICKELL,[2] M. ASCE and A. W. SMITH.[3]—The writers are most interested in the concept of reinforced concrete foundation ring described by the author, and are curious as to the real function of the ring. It would appear that the ring could serve one of two purposes; either—

(a) To stiffen the ground under the tank, or alternatively—
(b) To stiffen the weld between the wall of the tank and the floor by locally resisting rotation of the joint.

In case (a) it could be argued that, the deeper and stiffer the ring, then the larger the forces that it must resist, and hence the larger it must become. On the other hand, if no ring were provided, then no forces need be resisted; hence no ring would be necessary.

It is not clear why the ring should be rigid; the data tabulated from the bore logs would suggest that the soil is sufficiently strong to support the tank

a. Proc. Paper 1863, December, 1958, by Andrew M. Braswell, Jr.
1. Chf. Design Engr., McClelland Engrs., Inc., Houston, Tex.
2. Consulting Civil Engineer & Surveyor, Wellington, N. Z.
3. Associate, R. G. Brickell, Wellington, N. Z.

loadings, provided that the water test loading were applied in increments so controlled that general failure of the gumbo was avoided as the water level approached the top of the tank. It is suggested that a subsoil failure as in Fig. 2-c would occur only when all the soil beneath the tank had failed. Such is the theoretical conclusion. The writers would be interested in any data of actual failures of the form of Fig. 2-c.

For case (b), the function of the ring becomes more obvious. The fluid loading and soil reactions must impress some bending of the tank floor at the wall-to-floor joint. Here the rigid top of the ring would help by providing an anvil for the fluid pressure to hold the floor flat on the concrete. On the other hand, it has been found that quite economical stiffening may be obtained when the tank rests directly on the ground, by either thickening the floor plates at the joint, by welding in gusset plate stiffeners across the joint, or by a combination of both.

When the tank is raised well above the surrounding ground levels, as in Tank No. 7 in Fig. 7-A, there seems even less reason for using a ring. From the figure the ring is apparently retaining the sand fill beneath the tank. However, if the "fill sand" were extended beyond the tank some eight to ten feet and then allowed to fall off at its angle of repose, then there appears no reason for having the ring at all. The sand will merely retain itself. This form of tank foundation is not uncommon in New Zealand.

Would the author indicate whether the concrete ring is cast as continuous, or whether there are formed construction joints in it. A rough calculation would suggest that friction from the tank and internal soil loading would restrain the ring from contracting radially, and that concrete shrinkage could be a cause of ring breakage.

It is noted that the author has not included settlements of the middle of the floor in his tabulated data. Fig. 9 agrees with the writer's theoretical and field findings, that the centre of such a tank always settles much more than the edges, sometimes to the extent of twice the edge settlement. However a tank is gauged, the effect of the belly in the floor has perhaps a much more significant effect on the true volume than differential settlement of the edges.

The author's use of gunite lining for hot tanks is interesting and suggests that tanks described do not in fact settle further in the centre than at the edges. Any data on this subject would be welcome. Perhaps the foundation ring is the cause of this.

Finally, were any special steps taken to ensure that the soil in the bottom of the ring excavation remained undisturbed? Was a sand layer used, and was water excluded from the excavation?

A REVIEW OF THE ENGINEERING CHARACTERISTICS OF PEAT[a]

Discussions by T. Cameron Kenney and E. J. Zegarra

T. CAMERON KENNEY,[1] J. M. ASCE.—In many parts of the world there are areas where there is an abundance of pervious soils such as sands and gravels but where there is almost a complete absence of impervious soils such as clays and tills. In these areas, therefore, the design and construction of earth dams raises many interesting problems, one of which is what material to use for the impervious membrane. In Northern Canada and Scandanavia, these areas generally contain deposits of peat and this material has sometimes been used as impervious fill. The design of the dams is similar to the puddle clay, vertical core type of dam; the peat is placed to form a narrow central core and is supported on either side by banks of sand and gravel. In view of its impervious nature the peat performs quite satisfactorily as evidenced by the fact that some of these peat-core dams have been in successful operation for over two hundred years.(1)

With respect to the subject of shear strength of peat, this writer wishes to draw attention to reference number 2 which contains some data on vane tests in peat.

Mr. MacFarlane is to be thanked for publishing the results of his commendably complete review of published data concerning the engineering properties of peat.

REFERENCES

1. Treiten, A. A., 1956, "Anvendelse av torv i dammer." Norwegian Geotechnical Institute, Publication No. 14.
2. Skempton, A. W., and Henkel, D. J., 1953, "The Post-glacial Clays of the Thames Estuary at Tilbury and Shellhaven." Proc. 3rd Int. Conf. on Soil Mech. and Found. Eng., Vol. 1, ps. 302-308.

E. J. ZEGARRA,[2] A. M. ASCE.—A welcome addition to the knowledge of peat has been published by Mr. McFarlane and even more welcome is the effort advanced to establish a nomenclature. It is only by such means that transmitting information can be useful and reliable. Perhaps not all the terms and definitions are acceptable to everyone but having made them public invites elucidation and comment that tend to clarify the subject.

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a. Proc. Paper 1937, February, 1959, by Ivan C. MacFarlane.
1. Engr., H. G. Acres & Co. Ltd., Consulting Engrs., Niagara Falls, Canada.
2. Soils and Foundation Engr., The M. W. Kellogg Co., New York, N. Y.

During the course of a study for the industrial exploitation of a tropical island as a site for a refinery, peat was discovered. And because its extent, location and character were unique the peat became a crucial material for study and use. Peat exists in the island as a surface mantle varying from 30 to 40 inches thick and consists entirely of vegetal matter: roots from hair to fist thick. Beneath the peat the soil consists of a mixture of sand-size coral and shell fragments in a stratum 30 to 70 feet thick where decomposed, weathered and sound rock, successively, is found. The shell and coral debris is free of organic matter; the peat is free of mineral matter except of course where the two materials are in contact. To complete the description of the island it must be said that it is profusely covered by mangrove trees, both red and white species cover the entire surface; that the ground surface varies only a few inches from high to low points; that the vegetation is supported by the peat layer or conversely that the peat layer is the root system of the vegetation and finally that at high tide the surface is entirely submerged by a few inches and at low tide it emerges above the sea also by a few inches.

As suggested by the previous description the peat is a compressible and weak material partaking in these respects of the properties of natural sponges. Unless the vegetal fibres are broken the peat will regain its original volume after a compressive load has been removed and no restraint is placed on the rebound. These peculiarities make the application of tests for engineering properties somewhat vague: specific gravity, moisture content, saturation, compressive strength, consolidation acquire strange connotations.

Several techniques were tried to secure a good sample of the peat: block samples were found difficult to trim because cutting fibres required a tool as sharp as a shaving blade and as massive as an axe. With the best possible care a sample cut by hand lost so much water while being removed that its volume and water content were greatly altered. After much experimentation the most satisfactory sample was obtained by pushing a sharp edged shelby tube, 3 in. diameter into the peat at a very fast speed. A tube 20 in. long was pushed into the peat in a fraction of a second producing a sample about 16 in. long. This was the best recovery ratio and could not be improved because in the process of pushing the tube the fibres while being cut inevitably effected some compression of adjacent fibres. With these tubes however the water content could be determined more accurately and samples for consolidation and strength tests could be trimmed with a minimum of disturbance. The following results were obtained by standard tests:

Specific gravity of solids	2.1 (sample with some shell)
Per cent organic matter (ignition)	27.8 (sample with some shell)
Dry unit weight	7.8 - 8.3 pcf
Water content, per cent dry weight	719 - 795

A consolidation test on a sample 1 in. thick was carried out allowing each increment of load to remain for about 24 hours when it was doubled until the last load reached 3.5 tsf. Time-compression curves are shown on Fig. 1. It may be noticed that only at a load of 3.52 does the curve show a slight tendency to flatten; all preceding increments could have stayed on for a much longer period than 24 hours without significant change of slope in the curve. On Fig. 2 the pressure-void ratio curve is shown and beyond indicating an enormous void ratio change it could be likened to the curve of a very soft clay. Rebound was undoubtedly minimized by side friction.

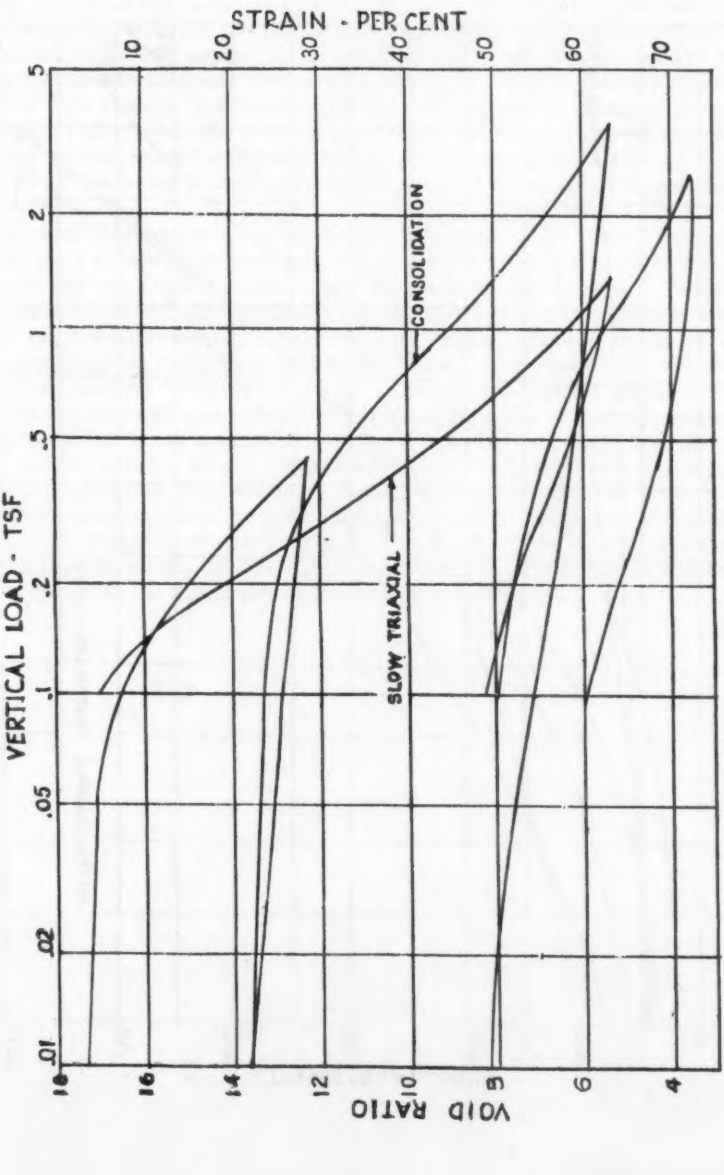
STRAIN - PER CENT
10
20
30
40
50
60
70
VERTICAL LOAD - TSF
5
2
1
.5
.2
.1
.05
.02
.01
VOID RATIO
18
16
14
12
10
8
6
4
CONSOLIDATION
SLOW TRIAXIAL

FIG. 1

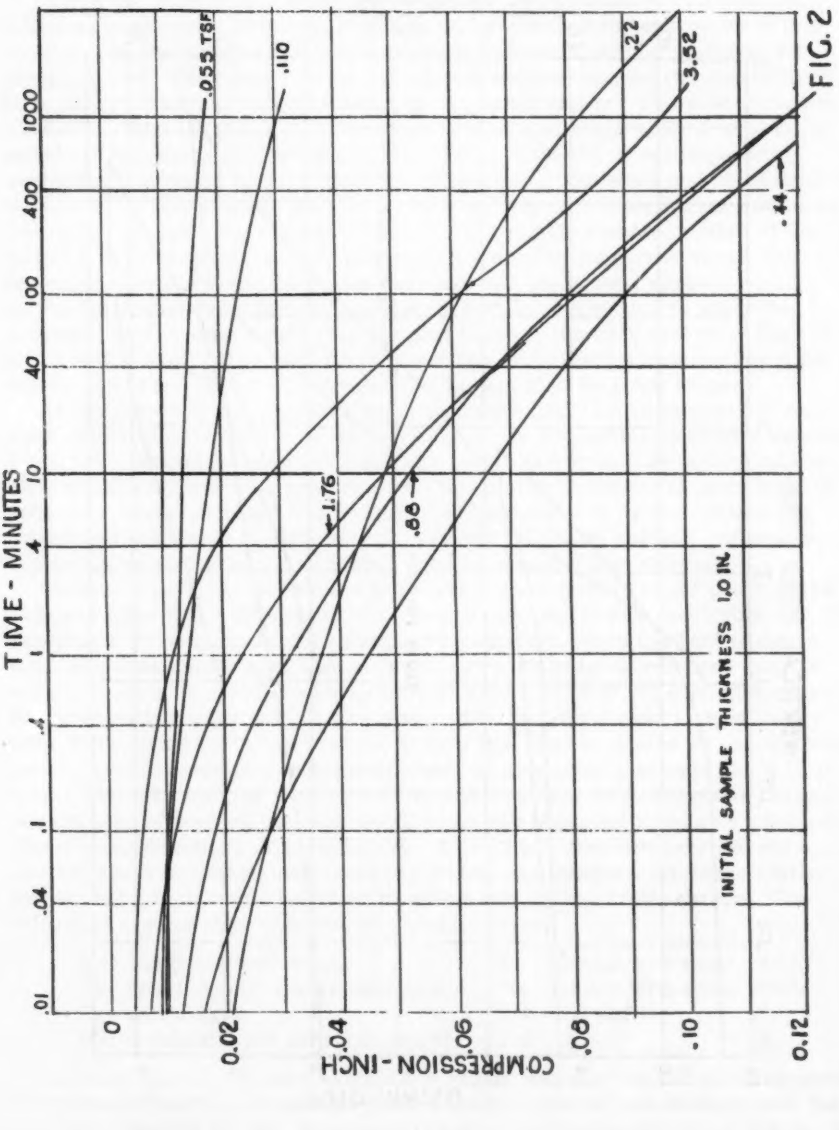
TIME - MINUTES
.01
.04
.1
.4
1
4
10
40
100
400
1000
COMPRESSION - INCH
0
0.02
.04
.06
.08
.10
0.12
.055 TSF
.110
.22
3.52
1.76
.88
.44
INITIAL SAMPLE THICKNESS 1.0 IN.

FIG. 2

On the same Fig. 2 the stress-strain curve of a slow triaxial test is shown. The similarity between the two curves is remarkable and if an arbitrary criterion for failure is applied i.e. a break in the curve, a compressive load of 0.1 tsf seems to be a fair measure of strength. Time-compression curves for the slow triaxial test were observed as longitudinal change and volume change. Volume was observed in a graduated pipette. Time-longitudinal change curves showed a definite flattening whereas volume change curves did not but on the basis of longitudinal change the increments of load were increased.

No other tests were made on this peat because at the time the problem was to estimate the probable settlement under a fill load of modest height. Further studies and tests will be made in the future to furnish data on moisture holding capacity, in-situ unit weight, absolute specific gravity, organic content.

In the definition of terms "percent ash, organic content" could be clarified by specifying the temperature of ignition since this would affect the organic "content" and the "percent" of ashes. The organic content reported in the previous paragraph was obtained by ignition on a bunsen burner flame. It is suggested that in the interest of uniformity the terms "mass specific gravity, unit weight, volume weight, Bulk density and Wet Density which are not wholly interchangeable be substituted by the term "Wet Unit Weight" (Mass Unit Weight) adopted by ASCE. Similarly "Dry Density" should be substituted by "Dry Unit Weight". Water Holding Capacity as defined by Mr. McFarlane is not as precise as the ASCE definition and as they both intend to measure the same property the ASCE definition should be preferred.

PROCEEDINGS PAPERS

The technical papers published in the past year are identified by number below. Technical-division sponsorship is indicated by an abbreviation at the end of each Paper Number, the symbols referring to: Air Transport (AT), City Planning (CP), Construction (CO), Engineering Mechanics (EM), Highway (HW), Hydraulics (HY), Irrigation and Drainage (IR), Pipeline (PL), Power (PO), Sanitary Engineering (SA), Soil Mechanics and Foundations (SM), Structural (ST), Surveying and Mapping (SU), and Waterways and Harbors (WW), divisions. Papers sponsored by the Department of Conditions of Practice are identified by the symbols (PP). For titles and order coupons, refer to the appropriate issue of "Civil Engineering." Beginning with Volume 82 (January 1956) papers were published in Journals of the various Technical Divisions. To locate papers in the Journals, the symbols after the paper number are followed by a numeral designating the issue of a particular Journal in which the paper appeared. For example, Paper 1859 is identified as 1859 (HY 7) which indicates that the paper is contained in the seventh issue of the Journal of the Hydraulics Division during 1958.

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JUNE: 1658(AT1), 1659(AT1), 1660(HY3), 1661(HY3), 1662(HY3), 1663(HY3), 1664(HY3), 1665(SA3), 1666(PL2), 1667(PL2), 1668(PL2), 1669(AT1), 1670(PO3), 1671(PO3), 1672(PO3), 1673(PL2), 1674(PL2), 1675(PO3), 1676(PO3), 1677(SA3), 1678(SA3), 1679(SA3), 1680(SA3), 1681(SA3), 1682(SA3), 1683(PO3), 1684(HY3), 1685(SA3), 1686(SA3), 1687(PO3), 1688(SA3)[c], 1689(PO3)[c], 1690(HY3)[c], 1691(PL2)[c].

JULY: 1692(EM3), 1693(EM3), 1694(ST4), 1695(ST4), 1696(ST4), 1697(SU2), 1698(SU2), 1699(SU2), 1700(SU2), 1701(SA4), 1702(SA4), 1703(SA4), 1704(SA4), 1705(SA4), 1706(EM3), 1707(ST4), 1708(ST4), 1709(ST4), 1710(ST4), 1711(ST4), 1712(ST4), 1713(SU2), 1714(SA4), 1715(SA4), 1716(SU2), 1717(SA4), 1718(EM3), 1719(EM3), 1720(SU2), 1721(ST4)[c], 1722(ST4), 1723(ST4), 1724(EM3)[c].

AUGUST: 1725(HY4), 1726(HY4), 1727(SM3), 1728(SM3), 1729(SM3), 1730(SM3), 1731(SM3), 1732(SM3), 1733(PO4), 1734(PO4), 1735(PO4), 1736(PO4), 1737(PO4), 1738(PO4), 1739(PO4), 1740(PO4), 1741(PO4), 1742(PO4), 1743(PO4), 1744(PO4), 1745(PO4), 1746(PO4), 1747(PO4), 1748(PO4), 1749(PO4).

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NOVEMBER: 1833(HY6), 1834(HY6), 1835(SA6), 1836(ST7), 1837(ST7), 1838(ST7), 1839(ST7), 1840(ST7), 1841(ST7), 1842(SU3), 1843(SU3), 1844(SU3), 1845(SU3), 1846(SU3), 1847(SA6), 1848(SA6), 1849(SA6), 1850(SA6), 1851(SA6), 1852(SA6), 1853(SA6), 1854(ST7), 1855(SA6)[c], 1856(HY6)[c], 1857(ST7)[c], 1858(SU3)[c].

DECEMBER: 1859(HY7), 1860(IR4), 1861(IR4), 1862(IR4), 1863(SM5), 1864(SM5), 1865(ST8), 1866(ST8), 1867(ST8), 1868(PP1), 1869(PP1), 1870(PP1), 1871(PP1), 1872(PP1), 1873(WW5), 1874(WW5), 1875(WW5), 1876(WW5), 1877(CP2), 1878(ST8), 1879(ST8), 1880(HY7)[c], 1881(SM5)[c], 1882(ST8)[c], 1883(PP1)[c], 1884(WW5)[c], 1885(CP2)[c], 1886(PO6), 1887(PO6), 1888(PO6), 1889(PO6), 1890(HY7), 1891(PP1).

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JANUARY: 1892(AT1), 1893(AT1), 1894(EM1), 1895(EM1), 1896(EM1), 1897(EM1), 1898(EM1), 1899(HW1), 1900(HW1), 1901(HY1), 1902(HY1), 1903(HY1), 1904(HY1), 1905(PL1), 1906(PL1), 1907(PL1), 1908(PL1), 1909(ST1), 1910(ST1), 1911(ST1), 1912(ST1), 1913(ST1), 1914(ST1), 1915(ST1), 1916(AT1)[c], 1917(EM1)[c], 1918(HW1)[c], 1919(HY1)[c], 1920(PL1)[c], 1921(SA1)[c], 1922(ST1)[c], 1923(EM1), 1924(HW1), 1925(HW1), 1926(PL1), 1927(HW), 1928(HW1), 1929(SA1), 1930(SA1), 1931(SA1), 1932(SA1).

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MARCH: 1960(HY3), 1961(HY3), 1962(HY3), 1963(IR1), 1964(IR1), 1965(IR1), 1966(IR1), 1967(SA2), 1968(SA2), 1969(ST3), 1970(ST3), 1971(ST3), 1972(ST3), 1973(ST3), 1974(ST3), 1975(ST3), 1976(WW1), 1977(WW1), 1978(WW1), 1979(WW1), 1980(WW1), 1981(WW1), 1982(WW1), 1983(WW1), 1984(SA2), 1985(SA2)[c], 1986(IR1)[c], 1987(WW1)[c], 1988(ST3)[c], 1989(HY3)[c].

APRIL: 1990(EM2), 1991(EM2), 1992(EM2), 1993(HW2), 1994(HY4), 1995(HY4), 1996(HY4), 1997(HY4), 1998(SM2), 1999(SM2), 2000(SM2), 2001(SM2), 2002(ST4), 2003(ST4), 2004(ST4), 2005(ST4), 2006(PO2), 2007(HW2)[c], 2008(EM2)[c], 2009(ST4)[c], 2010(SM2)[c], 2011(SM2)[c], 2012(HY4)[c], 2013(PO2)[c].

MAY: 2014(AT2), 2015(AT2), 2016(AT2), 2017(HY5), 2018(HY5), 2019(HY5), 2020(HY5), 2021(HY5), 2022(HY5), 2023(PL2), 2024(PL2), 2025(PL2), 2026(PP1), 2027(PP1), 2028(PP1), 2029(PP1), 2030(SA3), 2031(SA3), 2032(SA3), 2033(SA3), 2034(ST5), 2035(ST5), 2036(ST5), 2037(ST5), 2038(PL2), 2039(PL2), 2040(AT2)[c], 2041(PL2)[c], 2042(PP1)[c], 2043(ST5)[c], 2044(SA3)[c], 2045(HY5)[c], 2046(PP1), 2047(PP1).

JUNE: 2048(CP1), 2049(CP1), 2050(CP1), 2051(CP1), 2052(CP1), 2053(CP1), 2054(CP1), 2055(CP1), 2056(HY6), 2057(HY6), 2058(HY6), 2059(IR2), 2060(IR2), 2061(PO3), 2062(SM3), 2063(SM3), 2064(SM3), 2065(ST6), 2066(WW2), 2067(WW2), 2068(WW2), 2069(WW2), 2070(WW2), 2071(WW2), 2072(CP1)[c], 2073(IR2)[c], 2074(PO3)[c], 2075(ST6)[c], 2076(HY6)[c], 2077(SM3)[c], 2078(WW2)[c].

c. Discussion of several papers, grouped by divisions.

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NEWS OF THE SOIL MECHANICS AND FOUNDATIONS DIVISION OF ASCE

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JOURNAL OF THE SOIL MECHANICS AND FOUNDATIONS DIVISION
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DIVISION ACTIVITIES
SOIL MECHANICS AND FOUNDATIONS DIVISION
Proceedings of the American Society of Civil Engineers

NEWS

June, 1959

News from the West Coast

Considerable activity can be reported in the related fields of Soil Mechanics, Foundation Engineering, and Engineering Geology:

Visit of Dr. P. W. Rowe to University of California

During the month of April the University of California at Berkeley through the Department of Civil Engineering, presented a special series of twelve lectures on the "Design and Analysis of Anchored Bulkheads", by Dr. Peter W. Rowe, Reader in Civil Engineering at the University of Manchester, England and one of the world's leading authorities on this subject.

The subject matter covered by Dr. Rowe included consideration of cantilever, anchored, encastre and strutted walls in both clays and sands. Theoretical considerations were developed first; then experimental data derived from model studies were utilized to support the theoretical relationships. Effects of various wall materials were examined as well as flexibility of the soils in which the walls were embedded. Design procedures were formulated and illustrated by examples. The lecture series closed with consideration of work by Stroyer, Tschebotarioff and Hansen as well as a final appraisal of the present situation in Europe.

This lecture series was offered for credit to graduate students studying Soil Mechanics under Professor Harry B. Seed. Because of the unique opportunity afforded by the presence in this country of Dr. Rowe, the University extended a cordial invitation to practicing engineers throughout the San Francisco Bay Area, to participate.

On his way back to England, Dr. Rowe also presented lectures at The California Institute of Technology and Princeton University.

Meetings

On March 10th Professor K. B. Woods, President of The American Society for Testing Materials addressed a joint meeting of the Northern California District of A.S.T.M., The San Francisco Section ASCE, and The Structural

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Engineers' Association of Northern California. The subject of "Polar Construction" covered the effects of frost action and permafrost on the design and construction of highways, airfields, railroads and buildings in the Arctic and Sub-Arctic. The talk which was well illustrated by colored slides, covered projects in Alaska, Northern Canada, and Labrador and particular problems faced in construction of the Quebec, North Shore and Labrador Railroad.

On March 25th Professor James K. Mitchell of the University of California at Berkeley addressed the Soil Mechanics and Foundations Division of the San Francisco Section, ASCE on "Soil Technology—A New Branch of Soil Mechanics". Professor Mitchell described theoretical concepts and explained some of the phenomena governing basic soil behavior.

Organizations

A new organization, The California Association of Engineering Geologists has become active in the past year. Headquarters have been established in Sacramento with sections in Sacramento and Los Angeles which have now been joined by a San Francisco section. The various sections hold regular meetings on subjects of interest to Division members and publish a regular newsletter.

Terzaghi Lectures at University of Florida

Dr. Karl Terzaghi, Honorary Member of the American Society of Civil Engineers, visited the University of Florida from March 21 to 25 and presented two formal lectures sponsored by the Benton Memorial Lecture Fund. The first, entitled "Engineering Implications of Rock Weathering," was presented on March 23rd. In this lecture Dr. Terzaghi discussed the engineering importance of rock weathering, the agents responsible and the geographical areas where it may be expected. He cited examples of engineering problems associated with weathering and discussed their engineering solution. The following evening Dr. Terzaghi presented his second lecture entitled "Experiences in Regions of Granite Weathering" in which he discussed in detail the problems associated with the granite weathering at the site of the future Mammoth Pool Dam and the way in which engineers of Southern California Edison Company were able to take advantage of the weathered rock to provide the impervious material for the dam. Prior to this lecture Dr. Terzaghi was the dinner guest of the Gainesville Branch of the Florida Section of the ASCE.

During his visit Dr. Terzaghi also gave an informal talk to the University of Florida Student Branch of the ASCE in which he related some of his experiences and impressions regarding the personal attributes necessary to be successful in the different aspects of professional practice. He stressed the importance of engineering education which permits a man to better judge his limitations, and therefore permits him to act within his capabilities.

This was Dr. Terzaghi's first "vacation" in Florida and he took advantage of beautiful weather to enjoy Silver Springs, Marineland, St. Augustine, and the beaches on the Florida coast. He also went on several field trips to inspect limestone topography and unusual rock weathering phenomena in the Gainesville area.

The Twelfth Canadian Soil Mechanics Conference

The 12th Canadian Soil Mechanics Conference was held on December 8th and 9th, 1958 in Saskatoon. The meeting, sponsored by the Soil Mechanics Subcommittee of the Association Committee on Soil and Snow Mechanics, National Research Council, was arranged with the kind cooperation of the Prairie Farm Rehabilitation Administration and the University of Saskatchewan.

Earth dams was the topic on the first day's program. Mr. S. Ringheim, Project Engineer, South Saskatchewan River Dam, outlined the design and method of construction of this large dam. Mr. M. Peters, outlined the techniques used by P.F.R.A. to measure settlements and pore water pressures in dams under construction as well as after construction. Mr. C. F. Ripley, consultant, Vancouver, illustrated the complex geological conditions encountered in British Columbia and how these conditions affected site exploration and planning for dam sites. The fourth paper, by Mr. D. J. Bazett of Hydro-Electric Power Commission of Ontario, was concerned with pore water pressure measurements in a pumped storage reservoir at Niagara.

A half day of the conference was devoted to a panel discussion of the general problem of building on swelling and shrinking clays. Mr. C. B. Crawford of the Division of Building Research introduced the problem with some general comments on the characteristics of swelling and shrinking clays. He referred to work being done in other countries, the effect of climate, and gave a general evaluation of the problems with clays which are known to exist in Canada.

Dr. B. P. Warkentin of MacDonald College discussed the mechanism of volume change in clays, quoting the approximate magnitude of volume change possible in various pure clay systems. He outlined modern concepts concerning soil particle charges and the osmotic pressure concept in describing the swelling pressures of clays. Mr. M. Bozozuk of the Division of Building Research outlined his field research on ground movements in the "Leda clay" which occurs so extensively in Eastern Canada. He presented evidence that trees were a major factor in the differential shrinkage of this clay and reported both field and laboratory measurements to illustrate the magnitude and nature of the movements.

Professor A. Baracos of the University of Manitoba outlined generally the regions of swelling and shrinking clays in Western Canada and gave examples of common difficulties encountered in practice with standard foundations on these clays. Mr. L. Plotkin, architect, reviewed his experiences with the design, performance and economics of basementless houses on clay soils.

Mr. B. B. Torchinsky, consulting engineer, described his experience with the use of bored piles to prevent damage due to swelling clays. The design and economics of this type of foundation were discussed. There was much discussion concerning the depth of seasonal movements in swelling and shrinking clays, relationship between vertical and horizontal movements. It was concluded that the problem was serious in Canada and that more factual information and research were required.

Following the panel discussion, a paper by Mr. B. C. Laws described the pavement evaluation studies being carried out by the Saskatchewan Department of Highways using the Benkelman Beam. This paper and the lively discussion from the floor pointed to the need for more systematic studies such as seasonal moisture changes, moisture movement, density changes and frost action as

related to highway performance. How highway performance should be evaluated in addition to the Benkelman Beam test method was also discussed.

New Test Facility at Waterways Experiment Station

The U. S. Army Engineer Waterways Experiment Station at Vicksburg, Mississippi, which is the Corps of Engineers' principal agency for engineering research in the fields of hydraulics, concrete, flexible pavements, and soils, has recently been designated to conduct research aimed at developing rational means of improving the design of military vehicles from the standpoint of their ability to travel in soft soils and snow. This work will be conducted by the Army Mobility Research Center, a newly created branch of the Soils Division.

Construction of a test facility for this research is nearing completion. This facility, which will be the largest and best-equipped of its kind in the world, is housed in a building 200 ft. long by 80 ft. wide. Its main features are a plant for mixing soil and water to produce homogeneous soil at any given strength, and an apparatus for performing self-propelled and towed tests of scaled-down wheeled and tracked vehicles.

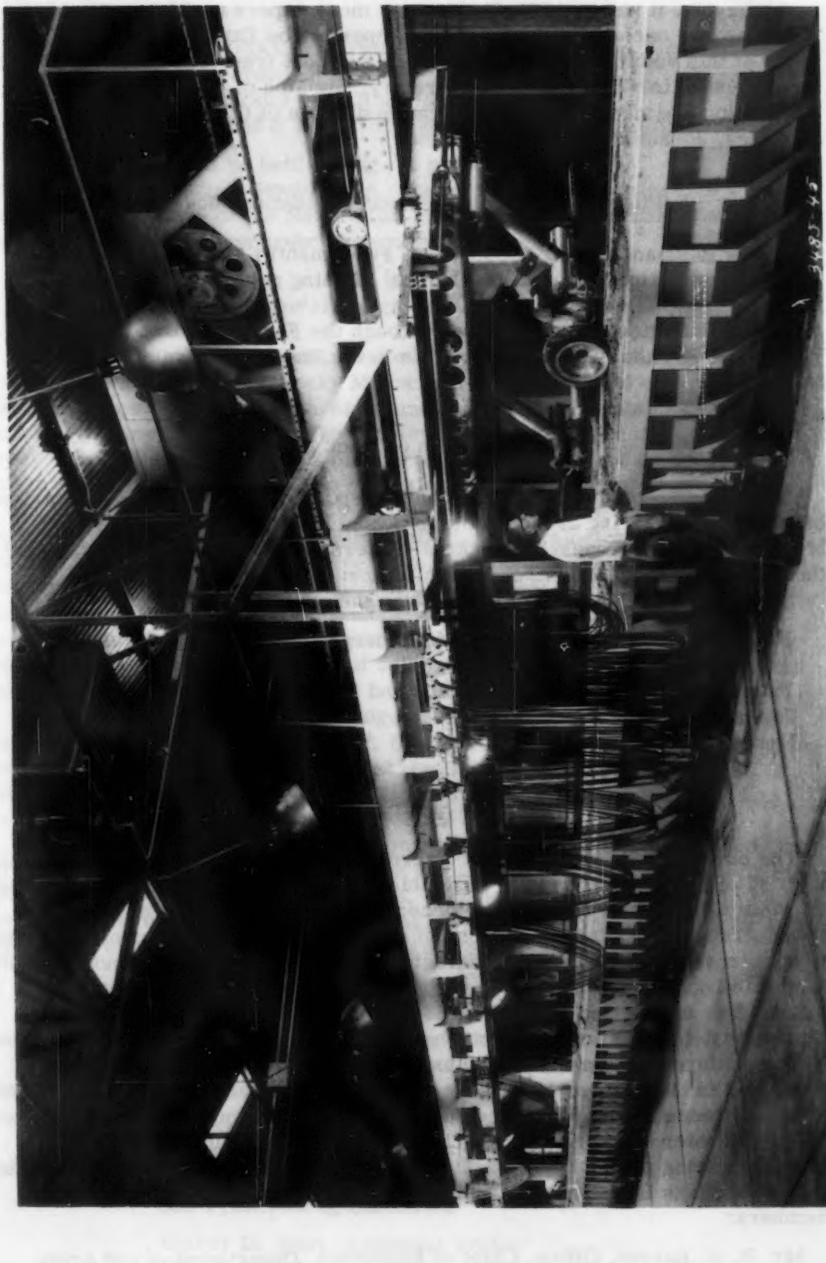
In the beginning stages of the research program, it is planned to conduct tests with conventional wheels and tracks over a wide range of soils and soil conditions, using the data accumulated to develop engineering relations between parameters of vehicle performance and soil condition. The fundamental relations expected to be developed will serve as a rational basis for the design and testing of new, and perhaps radically different wheel and track configurations for the improvement of vehicle performance in soft soils and snow.

The photo shows a test being made with a single towed wheel in a sandy soil. The maximum size wheel that can be tested is 34 in. outside diameter, equivalent to a 7.50-x-20 or 9.00-x-16 tire and wheel. The feasible maximum load is 3,500 lb, and the maximum speed capability (over the center one-third of the 165-ft-long test path) is 20 mph. Tracked-vehicle traction systems at scales not less than about 1:4 can be tested using the same basic apparatus by substituting the scaled track system for the wheel system shown.

South African Symposium on Expansive Clays

Since 1946 the problem of erecting structures on expansive soils has enjoyed a large amount of attention in South Africa and both theoretical and practical aspects of the problem have been investigated. During 1957 the following six papers on the subject were published in the Transactions of the South African Institution of Civil Engineers:

- Some Observations on the Movement of Buildings on Expansive Soils in Vereeniging and Odendaalsrus - L. E. Collins
- The Prediction of Total Heave from the Double Oedometer Test - J. E. Jennings and K. Knight
- The Time-Heave Relationship for Expansive Clays - J. A. De Wet
- Notes on the Design of Structures Founded on Heaving Soil - C. F. Templer
- Methods Used to Overcome Foundation Difficulties in Heaving Clays - C. P. Lange.
- The Design, Construction and Behaviour of a Reinforced Brick Building on Heaving Clay - V. R. Boardman.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In view of the widespread interest in many countries that this problem is now enjoying, it has been decided to bind these papers together with the discussions into one volume to form a "Symposium on Expansive Clays". The price of this volume will be £1.1.0 (sterling) post free. All inquiries should be addressed to—The Honorary Secretary, Division of Soil Mechanics and Foundation Engineering, South African Institution of Civil Engineers, P. O. Box 1183, Johannesburg, South Africa.

Russian Soil Mechanics and Foundations Journal

A new Russian Journal "Osnovanya, Fundamenti i Mehanika Gruntov" (Bases, Foundations and Soil Mechanics) is being published on a bimonthly basis.

This publication may be ordered through the Four Continent Book Corporation, 821 Broadway, New York. This is the Russian equivalent of Geotechnique.

Third Symposium on Rock Mechanics

The 3rd symposium on rock mechanics sponsored jointly by the Department of Mining Engineering at the Colorado School of Mines, the University of Minnesota and the Pennsylvania State University was held April 20-22, 1959 at the Colorado School of Mines, Golden, Colorado.

Sessions were devoted to the following themes:

Factors Common to Comminution, Underground Failure and Failures Resulting From Explosives
Factors Common to Soil Mechanics and Rock Failures
Seismology and Explosions
Nuclear Blasts in Mining

Research Conference on Shear Strength of Cohesive Soils

The purpose and scope of this conference are as follows: (1) The general purpose of the conference is to assemble, summarize, and discuss our present knowledge, or lack of knowledge, concerning the factors which govern the shear strength or failure conditions of cohesive soils. (2) The conference will be concerned with the strength characteristics of undisturbed, remolded, and compacted cohesive soils in both fully saturated and partially saturated states. The merits and limitations of available testing equipment and techniques for determination of the strength characteristics will also be discussed.

The conference as presently planned has been broadened from that mentioned in the October 1958 issue of the Newsletter. It will be a divisional-type conference sponsored by the Soil Mechanics and Foundations Division and open to all interested engineers. The conference, including the technical phases, is being planned by a Task Committee appointed by the Soil Mechanics and Foundations Division. The committee is made up of the following members:

Mr. R. A. Barron, Office, Chief of Engineers, Department of the Army
Dr. A. Casagrande, Harvard University

Dr. J. W. Hilf, U. S. Bureau of Reclamation, Department of the Interior, Secretary
Dr. M. J. Hvorslev, USAE Waterways Experiment Station, Vice-Chairman
Dr. R. B. Peck, University of Illinois
Dr. H. B. Seed, University of California
Dr. W. J. Turnbull, USAE Waterways Experiment Station, Chairman

The conference will be held during the period of June 13-17, 1960 at the University of Colorado at Boulder. A detailed plan of the conference will appear in a future issue of the Newsletter. Along with this detailed plan will be application blanks for conference attendance.

Rocky Mountain Region Soils Conference

A Soil Mechanics Conference, first of a planned series and also the first such conference held in the Rocky Mountain region, was held on April 23, 1959. The conference discussed engineering and architectural problems relating to shrinking and swelling soils. The problem is a great one in the Colorado Area. During the drought years of 1950 to 1954 the western states suffered millions of dollars damage due to shrinking and swelling soils.

Dr. R. V. Whitman and Dr. T. W. Lambe of Massachusetts Institute of Technology presented a paper dealing with the mineralogical, geological and engineering properties of expansive soils. Engineering problems associated with expansive clays were discussed by Professor R. E. Means of Oklahoma State University.

Mr. Chester McDowell, supervising soils engineer for the Texas highway department, spoke on the relation of soils testing to the design of pavements and structures.

Modern practices used in the design of foundations for structures on expansive clay were presented by R. F. Dawson, Professor of Civil Engineering and Associate Director of Research at the University of Texas. Mr. W. G. Holtz, Chief of the Earth Laboratory Branch of the U. S. Bureau of Reclamation, spoke on the engineering properties of expansive soil.

The one-day conference was co-sponsored by the Soil Mechanics and Foundations division of the Colorado Section of the American Society of Civil Engineers and the Colorado School of Mines. The conference's aim was to further develop accumulated knowledge of shrinking and expanding soils in a form that can be utilized readily by engineers and architects.

August Newsletter

Deadline date for arrival at this office of contributions for the August Newsletter: June 20, please.

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